Aharonov–Bohm oscillation of localization in antidot lattices

Seiji Uryu *, Tsuneya Ando

Institute for Solid State Physics, University of Tokyo 7-22-1 Roppongi, Minato-ku, Tokyo 106-8666, Japan

Abstract

Localization length in antidot lattices subjected to uniform magnetic fields is numerically calculated based on a finite-size scaling method. It is demonstrated that the localization length oscillates with a period of flux quantum $\Phi_0 = \hbar c/e$ as a function of flux passing through a unit cell near or higher than the critical magnetic field of an insulator–quantum Hall transition.

PACS: 72.15.Rn; 73.23.–b; 73.23.Ad

Keywords: Antidot lattice; Anderson localization; Insulator–quantum Hall transition

1. Introduction

Two kinds of quantum oscillations of resistivity were observed in hexagonal antidot lattices [1]. One is an oscillation with a period of $\Phi_0/2$ as a function of flux $\Phi$ passing through a unit cell near a zero magnetic field, where $\Phi_0 = \hbar c/e$ is the flux quantum. The other is that with a period $\Phi_0$ in magnetic fields near or higher than an insulator–quantum Hall transition point where the resistivity is observed to be independent of temperature. The former arises from an oscillation of the localization length [1,2]. The purpose of this paper is to study the $\Phi_0$ oscillation of the localization length and an insulator–quantum Hall transition in a hexagonal antidot lattice by a numerical finite-size scaling method.

2. Model and method

As a model of antidot lattices, we take a two-dimensional array of quantum-wire junctions, each of which is characterized by a scattering matrix calculated in a nearest-neighbor tight-binding model [2,3]. We use the following model potential $V(r)$ of an antidot in a hexagonal lattice with a center at the origin [4,5]:

$$V(r) = U_0 \left| \cos \left( \frac{\pi a_1 \cdot r}{a^2} \right) \cos \left( \frac{\pi a_2 \cdot r}{a^2} \right) \right|^{4\beta/3},$$

with $U_0$ being a maximum of the potential, $a$ a potential period, $a_1 = (\sqrt{3}a/2, a/2)$, $a_2 = (0, a)$, and $\beta$ a parameter describing the steepness of the potential.

We consider two kinds of disorders. One is the impurity potential and the other is the fluctuation in the
antidot diameter \( d \) [4,5]. The former is characterized by the mean free path \( l_e \) of the two-dimensional system in the absence of the antidot potential. The latter is characterized by \( d_f = \sqrt{\langle (d - \langle d \rangle)^2 \rangle} \), where \( \langle \cdots \rangle \) means an average over different antidots.

We shall use a finite-size scaling method [6,7]. The inverse localization length \( \lambda(\lambda) \) is calculated for a long strip-form antidot lattice with a finite width \( L \). We assume a scaling hypothesis for \( \lambda(L) \):

\[
\lambda(L) = f(\lambda_{\text{2D}}),
\]

(2.2)

where \( \lambda_{\text{2D}} \) is an inverse localization length in a two-dimensional system. It should be noted that the system in the absence of a magnetic field, i.e., \( \Phi/\Phi_0 = 0 \), may have to be excluded because it belongs to a different universality class (orthogonal) from that of the others (unitary).

In the following, we choose the parameters \( \lambda_F/a = 0.27 \) with \( \lambda_F \) being the Fermi wavelength, \( l_e/a = 4 \), \( d_f/a = 0.035 \), and \( \beta = 1 \) which are consistent with those of experiments [1,4,5] and use \( d/a = 0.8 \) which will later turn out to be slightly larger than that in actual systems. The width of a strip system is chosen as \( L/\sqrt{3}a = 8, 12, \) and 16, and periodic boundary conditions are imposed in the direction of the width.

The typical length of the strip system is 20,000 in units of \( a \) for \( 0 \leq \Phi/\Phi_0 \leq 6 \) and 10 \leq \Phi/\Phi_0 \leq 12 \) in the strong-localization regime and 120,000 for 6.25 \leq \Phi/\Phi_0 \leq 9.75 in the weak localization regime. The typical statistical errors of the resulting \( \lambda(L) \) for the strong and the weak localization regimes are less than 4% and 2.5%, respectively.

3. Results

Figs. 1(a)–(c) show the system-size dependence of normalized inverse localization length \( \lambda(L) \) at various magnetic fields lying in the range \( 0 \leq \Phi/\Phi_0 \leq 12 \). The data with large values and the strong dependence on the size \( L/a \) indicate the strong localization regime and the data with small values and the weak dependence on the size indicate the weak localization regime.

The qualitative feature in Fig. 1 is that the localization is strongest at a zero magnetic field, becomes weaker with increasing magnetic field, and becomes strong again after taking a minimum at \( \Phi/\Phi_0 \sim 8 \) where the normalized inverse localization length has almost no dependence on the system size. It is likely that states are extended and an insulator–quantum Hall transition occurs at this magnetic field as will be discussed below.

The finite-size scaling hypothesis given by Eq. (2.2) means that each curve of Fig. 1 should be reduced to a
part of a single common curve by a shift in the horizontal direction [6,7]. This shift corresponds to a change of the length scale $a$ and the resulting scale is proportional to the localization length in a two-dimensional system. An absolute value of the inverse localization length, which cannot be obtained by the finite-size scaling, can be determined by the condition that the geometric average of the inverse localization length near a zero magnetic field reproduces that of the previous results obtained in a Thouless-number method which provides an absolute value [2].

Fig. 2 shows the scaling function determined by a scale change such as that mentioned above. It shows that the inverse localization length at different magnetic fields is scaled into a single common curve and the finite-size scaling (2.2) works quite well.

Fig. 3 shows the resulting inverse localization length. Near a zero magnetic field $0 \leq \Phi/\Phi_0 \leq 1$, the inverse localization length oscillates with a period $\Phi_0/2$. This Al'tshuler–Aronov–Spivak-type oscillation can be clearly seen in the inset which gives a blowup in this region and explains existing experiments [1]. In the inset, the previous Thouless-number results [2] are also shown. The results obtained in different methods are in good agreement with each other.

There are two important features in higher magnetic fields. One is the presence of a minimum in the inverse localization length at $\Phi/\Phi_0 \approx 8$ as expected from the results in Fig. 1. The localization length at this field exceeds 5000 in units of $a$ ($\sim 1$ mm for typical $a = 2000 \text{ Å}$). Although it is quite difficult to draw a definite conclusion whether states are really extended or not, we can practically regard states to be extended at this field and the corresponding field as an insulator–quantum Hall transition point. Another important feature is the oscillation with a period $\Phi_0$ in magnetic fields close to or larger than this critical magnetic field. In this case, the localization effect becomes weaker at magnetic fields where $\Phi/\Phi_0$ is exactly an integer.

The present results are qualitatively in good agreement with those of experiments [1]. In the experiments, a prominent oscillation with a period $\Phi_0$ is observed in magnetic fields near or higher than the magnetic field corresponding to an insulator–quantum Hall transition where the resistivity is independent of temperature. In the oscillation, the diagonal conductivity takes peaks at integer $\Phi/\Phi_0$ in consistency with the decrease of the localization effect obtained here theoretically.
There seems to be some quantitative difference. The critical magnetic field of the experiments is $\Phi/\Phi_0 \sim 4$ which is smaller than our result $\Phi/\Phi_0 \sim 8$. This difference may be attributed to a difference in the value of $d/a$, i.e., the actual $d/a$ in experiments seems to be smaller than $0.8$ adopted in the numerical calculation. Preliminary calculations for $d/a = 0.7$ currently underway seem to give a critical magnetic field close to that obtained experimentally.

It should be noted that the scaling of data near the critical magnetic field must be carefully performed due to the presence of a possible correction to scaling [8]. More accurate analysis including such corrections requires a further reduction in statistical errors and calculations for wider systems. Such works are underway.

4. Summary and conclusion

The localization effect in hexagonal antidot lattices has been numerically studied with the use of a finite-size scaling method. It has been shown that the inverse localization length takes a minimum at a certain critical magnetic field and oscillates with a period $\Phi_0$ around this critical field. The localization length at the critical field exceeds 5000 in units of the array period and is therefore macroscopically large. It is likely that this field corresponds to an insulator–quantum Hall transition.

Acknowledgements

This work is supported in part by the Japan Society for the Promotion of Science (“Research for the Future” Program JSPS-RFTF96P00103). One of the authors (S.U.) has been supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists. Some numerical calculations were performed on FACOM VPP500 in Supercomputer Center, Institute for Solid State Physics, University of Tokyo.

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