Aharonov–Bohm oscillation and periodic orbits in antidot lattices

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Abstract

The magnetoresistance of square antidot lattices is studied numerically by systematically varying the aspect ratio and the form of the antidot potential. The appearance of the Aharonov–Bohm (AB) type oscillation is shown to be strongly correlated with the presence of a particular periodic orbit encircling an antidot.

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1. Introduction

A fine oscillation is observed in the magnetoresistance of antidot lattices near the commensurate magnetic field given by $2R_c = a$ with $R_c$ being the classical cyclotron radius and $a$ a lattice constant [1,2]. This oscillation is often called Aharonov–Bohm (AB) type oscillation because the period is about the flux quantum $\Phi_0 = \hbar/e$ as a function of the flux $\Phi$ passing through a unit cell. The purpose of this paper is to demonstrate a strong correlation between the AB type oscillation and a particular periodic orbit encircling an antidot.

There are various possible mechanisms causing this AB type oscillation. Energy levels obtained by a semiclassical quantization of a periodic orbit encircling an antidot may lead to peaks in the density of states [2]. A quasi-periodic change in the band structure with period $\Phi_0$ related to Hofstadter’s butterfly spectra [3] is also a candidate. Both of these mechanisms can cause quantum oscillation of the transport coefficients [4–6].

In this paper, we study the correlation between the presence of periodic orbits encircling an antidot and the AB type oscillation by varying the antidot potential systematically. It is shown that the disappearance of periodic orbits and the AB type oscillation are correlated to each other, confirming that the AB type oscillation is due to semiclassically quantized energy levels associated with the periodic orbit.

2. Model and method

We replace a square antidot lattice by a quantum-wire junction array characterized by a scattering matrix which is calculated in a nearest-neighbor tight-binding model [7]. We use the following model potential $V(r)$ of an antidot with its center at the origin,

$$V(r) = E_F \left( \frac{d/2 + A - r}{A} \right)^2 \theta(d/2 + A - r),$$

(2.1)
where $E_F$ is the Fermi energy, $d$ is a diameter of an antidot, $r$ is the distance from the center, and $\theta(t)$ is the step function defined by $\theta(t) = 1$ for $t > 0$ and 0 for $t < 0$. The parameter $\Lambda$ describes the steepness of the antidot potential. We consider the case $r_E/a = 3.77$ where $r_E$ is the Fermi wavelength. This corresponds to the electron concentration $n_s = 2.2 \times 10^{11}$ cm$^{-2}$ for $a = 2000$ Å which is comparable to $n_s = 1.4 \sim 2.8 \times 10^{11}$ cm$^{-2}$ in the experiments [2]. For this parameter, we have $\Lambda/a \approx 0.24$ through comparison with self-consistent calculations [8,9]. The typical channel number, i.e., the number of one-dimensional subbands below the Fermi level in the narrow region between adjacent antidots, is two in this system.

Effects of impurity scattering are taken into account by introducing short-range scatterers whose strength is characterized by the mean free path $l_e$ of the two-dimensional system in the absence of the antidot potential. We use $l_e/\pi a = 2$ throughout this paper. The transmission probabilities of finite-size systems are averaged over impurity configurations and are used for the calculation of the resistance.

3. Results

Fig. 1 shows the calculated results of the magnetoresistance of four terminal antidot lattices as schematically shown in the left inset of Fig. 2 for various values of the aspect ratio $d/a$ between the antidot diameter and a lattice constant. Short vertical lines indicate the magnetic fields where the quantized levels of a periodic orbit encircling an antidot cross the Fermi energy. Solid lines mean stable orbits and dotted ones unstable. The

![Fig. 1](image1.png)

**Fig. 1.** Calculated magnetoresistance for $\Lambda/a = 0.24$. The origin of the magnetoresistance is shifted for each aspect ratio.

![Fig. 2](image2.png)

**Fig. 2.** Critical values of the aspect ratio at which the periodic orbit encircling an antidot disappears. $\Lambda/a = 0.18$ (solid line), 0.24 (dotted line), and 0.36 (dashed line). The right upper and lower insets show the periodic orbit (solid line) giving the AB type oscillation and another orbit (dotted line) that is unstable. These orbits merge and disappear when the aspect ratio approaches a critical value. The left inset shows a schematic illustration of a four terminal antidot lattice used in calculations.
AB type oscillation changes as a function of the magnetic field in an irregular manner, but the dependence on the field is, on the whole, in good agreement with that of the position of quantized levels of the periodic orbit. The oscillation disappears or becomes unrecognizable near the field corresponding to $2R_c = a$ for large aspect ratio $d/a = 0.7$. This field corresponds to the disappearance of the periodic orbit. In fact, the periodic orbit is absent in the hatched region.

Fig. 2 shows the boundary of the existence of the periodic orbit encircling an antidot. The insets in the right hand side of the figure show the periodic orbit (solid line) for $d/a = 0.6$ and 0.677. With the increase of $d/a$ it merges into another orbit (dotted line), which is unstable and has a longer trajectory, and disappears. Fig. 2 contains the boundary in the case of $\Delta/a = 0.18$ and 0.36 as well. The critical aspect ratio becomes smaller with the increase of $\Delta/a$, i.e., as the antidot potential becomes broader.

Fig. 3 shows the calculated results for a steeper potential $\Delta/a = 0.18$. There is a tendency that the AB type oscillation persists with the decrease of the field for $d/a = 0.7$ even after the disappearance of the periodic orbit. Fig. 4 shows the results for a broader antidot potential $\Delta/a = 0.36$. The AB type oscillations becomes almost unrecognizable before the periodic orbit disappears.

The present antidot lattices are in a full-quantum regime rather than in a semi-classical regime because of the presence of only a few channels between neighboring antidots. The results show that they still retain to some extent characteristic features corresponding to semi-classical energy levels of the periodic orbit encircling an antidot in their energy spectrum. In antidot lattices with a steeper potential, the effective kinetic energy of an electron is larger and semiclassical features are expected to be more robust. In lattices with a broader potential, on the other hand, quantum effects are expected to be more important due to lowering of the
kinetic energy. The present results are consistent with such a picture.

4. Summary

The relation between the AB type oscillation and a particular periodic orbit encircling an antidot in square antidot lattices has been studied by systematically varying the aspect ratio and the form of the potential. The disappearance of the AB type oscillation is correlated quite well with the disappearance of the periodic orbit if we consider the change of quantum effects depending on the steepness of the antidot potential.

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