Effects of symmetry breaking on perfect channel in metallic carbon nanotubes

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Abstract. Numerical calculations of the conductance are performed to study symmetry breaking effects on a perfectly conducting channel in metallic carbon nanotubes existing unless the potential range of scatterers is smaller than the lattice constant. The perfect channel is fragile against perturbations when there are several bands at the energy, while it is robust in the energy range of metallic linear bands.

Transport properties of carbon nanotubes are extremely interesting. It was shown that in metallic nanotubes there is no backscattering even in the presence of scatterers unless their potential range is smaller than the lattice constant of a two-dimensional graphite [1]. When there are several bands at the Fermi level, interband scattering appears, but it has been shown that a perfectly conducting channel transmitting through the system without being scattered back exists and the conductance is always larger than $2e^2/\pi h$ [2].

In this paper we consider effects of various kinds of perturbations breaking the symmetry leading to the unique transport properties and show that the perfect channel is very fragile against such perturbations when there are several bands at the energy, while it is much more robust in the energy range of metallic linear bands.

In a graphite sheet the conduction and valence bands consisting of $\pi$ states cross at K and K’ points of the Brillouin zone. Electronic states near a K point are described by the \( k \cdot p \) equation [2]. In a metallic nanotube, states can be obtained by imposing periodic boundary conditions in the circumference direction \( \mathbf{F}(\mathbf{r} + \mathbf{L}) = \mathbf{F}(\mathbf{r}) \). Then, the energy bands are given by \( \varepsilon_n(k) = \pm \gamma \sqrt{n^2 + k^2} \), where \( n = 0, \pm 1, \cdots, \kappa(n) = 2\pi n/L \), \( k \) is the wave vector along the axis, and + and − for the conduction and valence bands, respectively. They are shown in Fig. 1.

This equation is the same as that of a massless neutrino and possesses a topological singularity at \( k = 0 \) giving rise to Berry’s phase under a rotation around the origin in the wave-vector space [3]. It has special time reversal symmetry which leads to various exotic transport properties mentioned above if being combined with Berry’s phase. This symmetry is destroyed obviously by a magnetic field \( H \) perpendicular to the axis and a flux \( \phi \) due to a magnetic field parallel to the axis. The dimensionless quantity characterizing the strength is \( (L/2\pi \ell)^2 \) for the magnetic field and \( \phi/\phi_0 \) for the flux, where \( L = |\mathbf{L}|, \ell = \sqrt{\hbar/eH} \), and \( \phi_0 = \hbar c/e \).

The symmetry is affected also by scatterers with range smaller than the lattice constant \( a \). Its strength is characterized by \( \delta \) which denotes the amount of short-range scatterers relative to the total amount of scatterers \( (0 < \delta < 1) \). It is destroyed also by the presence of trigonal warping of the bands around the K and K’ points. The effect of warping can be described by higher order \( k \cdot p \) terms [4] and is characterized by \( \beta a/L \) where \( \beta \) is a dimensionless quantity of the order of unity.

For actual calculations, we consider a system with randomly distributed scatterers with strength \( \pm u \) and con-
centrations $n_i$. The scattering strength is characterized by the dimensionless quantity $W = n_i u^2 / 4\pi\gamma^2$. Using numerical results of the conductance as a function of the length $A$, we can determine the localization length in the presence of such symmetry breaking perturbations. In fact the geometric average of the conductance is $\bar{G} \propto \exp(-2\alpha A)$ for nanotubes with sufficiently large $A$ where $\alpha$ is the inverse localization length.

Results for several values of the energy $\varepsilon$ are shown in Fig. 2. The number of the bands are 1, 3, 5, and 7 for $\varepsilon(2\pi\gamma/L)^{-1} = 0.5, 1.5, 2.5$, and 3.5, respectively. When there are several bands at the Fermi level, the inverse localization length exhibits a near-plateau behavior above a small critical value. The plateau values are nearly same in all the cases and given approximately by the inverse of the sum of the mean free path of the bands [2] at the energy. When the energy is in the range of metallic linear bands, the inverse localization length is much smaller unless the symmetry breaking parameter becomes appreciable. Therefore, the perfect channel can easily be destroyed even by a very small perturbation when there are several bands at the energy, while it is very robust in the energy range of metallic linear bands.

It should be noted that various effects can be incorporated as an effective magnetic flux such as the nonzero curvature present in actual nanotubes [5, 6] and the lattice distortion [6, 7, 8]. It is highly likely, therefore, that an effective flux of the order of $\phi/\phi_0 \sim 10^{-2}$ is always present in actual nanotubes. It turns out further that effects of the trigonal warping are surprisingly strong among those symmetry breaking effects. Even considering the ambiguity in the exact value of $\beta$, we can expect that the effective strength of the trigonal warping is well in the near-plateau region for typical tubes. In actual nanotubes, therefore, the perfectly conducting channel is likely to be destroyed almost completely when there are several bands at the energy. The absence of backward scattering when the Fermi level lies in the region of metallic linear bands remain quite robust against various symmetry breaking effects, however.

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**REFERENCES**