Resistivity of Two-Dimensional Systems with Electrostatic and Magnetic-Field Modulations

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The resistivity determined by electron-electron scattering is studied in two-dimensional systems with weak periodic electrostatic and magnetic-field modulations. Energy gaps opening up at the boundary of the Brillouin zone play an essential role in determining the modulation dependence of the resistivity. The results of numerical calculations are in good agreement with recent experiments.

Keywords: Umklapp electron-electron scattering, surface lateral superlattice, magnetic-field modulation

1. Introduction

In periodic systems electron-electron scattering can contribute to the resistivity due to Umklapp processes which do not conserve the total electron momentum. This resistivity component was observed in two-dimensional (2D) systems with weak one-dimensional periodic modulations [1-4]. There are two types of modulations, electrostatic modulation [1] and magnetic-field modulation [2-4]. In usual systems fabricated on a GaAs/Al_xGa_{1-x}As heterostructure with high mobility, energy broadening due to potential disorder is smaller than dominant energy gaps due to periodic modulations [3,4]. The purpose of this paper is to explore effects of electron-electron scattering on the resistivity in such modulated 2D systems.

2. Model and Method

We consider 2D electron systems with weak periodic modulations and take into account effects of the Coulomb interaction in the lowest order perturbation. Since the magnetic-field modulation is usually realized by depositing magnetic materials on the surface, it involves an inevitable electrostatic modulation due to deformation of lattice. Therefore, we consider electrostatic and magnetic-field modulations with the same period on the 2D xy plane such as

$$\begin{aligned} V(x) &= V_0 \cos(gx), \\ B_z(x) &= B_0 \cos(gx + \delta), \end{aligned} \quad (2.1)$$

respectively, where V_0 and B_0 are the amplitude of each modulation and δ is their phase difference. The total modulation strength becomes

$$|V_t| = \left| \frac{V_0}{2} + i \frac{\hbar \omega_c k_y}{2g} e^{-i\delta} \right|, \quad (2.2)$$

with $\omega_c = eB_0/m^*$. The wave function for a noninteracting system is written as

$$\Psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{\Omega}} \sum_{\mathbf{G}} C_{\mathbf{G}}(\mathbf{k}) e^{i(\mathbf{k}+\mathbf{G}) \cdot \mathbf{r}}, \quad (2.3)$$

where Ω is the system area and $\mathbf{G} = n\mathbf{g}$ with $\mathbf{g} = (g, 0)$ and $n = 0, \pm 1, \dots$.

In the following we shall concentrate ourselves to the case $g < 2k_F$ for which energy gaps on the Fermi surface open up at boundaries of Brillouin zone. In a weak modulation limit, dominant energy gaps are those

at the boundary of the first Brillouin zone, i.e., $k_x = \pm g/2$ and $k_y = \pm k_g = \pm \sqrt{k_F^2 - g^2/4}$ and the gap width becomes $|V_t|$. The Fermi surface depends on δ in Eq. (2.1) in the presence of both electrostatic and magnetic-field modulations. For example, the Fermi surface for $\delta = \pi/2$ is asymmetric regarding the k_x axis and energy gaps close off for negative k_y when $V_0 = \hbar \omega_c k_g/g$, while it is always symmetric for $\delta = 0$.

The wave function in a weak modulation limit is approximately given by three plain waves with $\mathbf{k}$ and $\mathbf{k} \pm \mathbf{g}$. Then $C_0(\mathbf{k})$ is always of the order of unity, while $C_{\pm \mathbf{g}}(\mathbf{k})$ is of the first order of $|V_t|$ except in the energy-gap region in the vicinity of $k_x = \mp g/2$.

We calculate the resistivity based on a Boltzmann transport equation with a constant relaxation time. The use of a relaxation time independent of the $\mathbf{k}$ direction is semi-quantitatively valid as long as impurity scattering is dominant and effects of electron-electron scattering can be regarded as small corrections, because in this case the relaxation is nearly isotropic along the Fermi line [5]. It gives also an upper limit of the resistivity.

In this approximation the resistivity due to electron-electron scattering $\Delta\rho$ is given by [6]

$$\begin{aligned} \Delta\rho &= \tilde{C} \left(\frac{e^2}{\lambda_F E_F} \right)^2 \left(\frac{k_B T}{E_F} \right)^2 \frac{h}{e^2}, \\ \tilde{C} &= \frac{1}{48} \sum_{\Delta \mathbf{G}} \frac{2}{(2\pi)^2 \hbar} \int d\mathbf{q} \frac{(\Delta \mathbf{v} \cdot \mathbf{u})^2 W_{\Delta \mathbf{G}}}{|\mathbf{v}_1 \times \mathbf{v}_3| |\mathbf{v}_2 \times \mathbf{v}_4|}, \end{aligned} \quad (2.4)$$

where $\tilde{C}$ is a dimensionless coefficient, λ_F the Fermi wave length, E_F the Fermi energy, k_B the Boltzmann constant, T a temperature, $\mathbf{q} = \mathbf{k}_3 - \mathbf{k}_1 = \mathbf{k}_2 - \mathbf{k}_4 - \Delta \mathbf{G}$ with $\Delta \mathbf{G} = \mathbf{G}_3 + \mathbf{G}_4 - \mathbf{G}_1 - \mathbf{G}_2$, $\Delta \mathbf{v} = \mathbf{v}_3 + \mathbf{v}_4 - \mathbf{v}_1 - \mathbf{v}_2$, $\mathbf{u}$ a unit vector along the electric field, and $W_{\Delta \mathbf{G}}$ the transition probability from $\mathbf{k}_1, \mathbf{k}_2$ to $\mathbf{k}_3, \mathbf{k}_4$. The summation is taken over sets of $\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3$, and $\mathbf{G}_4$ which satisfy $\Delta \mathbf{G} = n\mathbf{g}$ with $n = 0, \pm 1, \dots$. The integral includes summation over possible sets of $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$, and $\mathbf{k}_4$ for given $\mathbf{q}$. The processes with $\Delta \mathbf{G} = 0$ are called normal process and those with $\Delta \mathbf{G} \neq 0$ Umklapp process. In the following $\Delta\rho_{xx}$ is considered and $\Delta\rho_{yy}$ is neglected completely because $\Delta\rho_{yy}$ is much smaller than $\Delta\rho_{xx}$ due to the fact $\Delta v_x \gg \Delta v_y$ [7].

The symmetry of the space part of the wave function with respect to exchange of two particles is taken into

account by using the following transition probability:

$$W_{\Delta\mathbf{G}} = \frac{2\pi}{\hbar} \frac{\pi^4 \hbar^4}{m^{*2}} (|A|^2 + |B|^2 + |A - B|^2),$$

$$A = \sum_{\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3, \mathbf{G}_4} \frac{k_F V_{ee}(q')}{2\pi e^2} C_{\mathbf{G}_4}^*(\mathbf{k}_4) C_{\mathbf{G}_3}^*(\mathbf{k}_3) C_{\mathbf{G}_1}(\mathbf{k}_1) C_{\mathbf{G}_2}(\mathbf{k}_2), \quad (2.5)$$

where $V_{ee}(q')$ is an effective screened Coulomb interaction including effects of nonzero thickness of 2D plane [8] and $\mathbf{q}' = \mathbf{k}_3 - \mathbf{k}_1 + \mathbf{G}_3 - \mathbf{G}_1$. The term B is obtained by setting $\mathbf{q}' = \mathbf{k}_4 - \mathbf{k}_1 + \mathbf{G}_4 - \mathbf{G}_1$ in the expression of A .

As the wave function is superposition of plane waves with $\mathbf{k}$ and $\mathbf{k} \pm \mathbf{g}$ as mentioned before, two-particle scattering from $\mathbf{k}_1, \mathbf{k}_2$ to $\mathbf{k}_3, \mathbf{k}_4$ consists of various scatterings between plane waves with $\mathbf{k}_i + \mathbf{G}_i$ where $i = 1, 2, 3$, and 4 and $\mathbf{G}_i = 0$ and $\pm \mathbf{g}$. Normal scattering between plane waves with $\mathbf{k}_i$, i.e., $\mathbf{G}_i = 0$, is most probable because $C_0(\mathbf{k})$ is always of the order of unity. However, contribution of normal process is small as compared to that of Umklapp process because the total velocity transfer $\Delta \mathbf{v}$ is small in most cases.

Lowest order Umklapp processes with $\Delta \mathbf{G} = \pm \mathbf{g}$ consist of scattering with a plane wave with $\mathbf{k}_i \pm \mathbf{g}$ and the other three waves with $\mathbf{k}_j$ where $j \neq i$ because they involve a single coefficient $C_{\pm \mathbf{g}}(\mathbf{k}_i)$. Consider the case that $\mathbf{G}_1 = \pm \mathbf{g}$ and $\mathbf{G}_j = 0$ for $j = 2, 3$, and 4, for example. When $\mathbf{k}_1$ lies outside energy-gap regions, the transition probability is proportional to $|V_t|^2$ because $C_{\pm \mathbf{g}}(\mathbf{k}_1)$ is of the first order of $|V_t|$. When $\mathbf{k}_1$ is inside an energy-gap region, on the other hand, the matrix element is independent of V_t . Because of the phase-space restriction imposed on $\mathbf{k}_1$, the contribution of such processes to the resistivity becomes roughly proportional to $|V_t|$. Therefore, processes with the smallest momentum transfer $\hbar \mathbf{g}$ and with at least one state in an energy-gap region become dominant.

3. Numerical Results

Two representative cases with large g ($= 1.2k_F$) and small g ($= 0.12k_F$) are calculated. The former corresponds to a weak modulation limit and the latter to typical experiments in which not only the lowest order but also higher order energy gaps are important. At a GaAs/Al_xGa_{1-x}As heterostructure, the effective mass is $m^*/m \sim 0.067$, the static dielectric constant is $\kappa \sim 13.2$, and $2\pi/q_{TF} \sim 300$ Å with q_{TF} being the Thomas-Fermi screening wave vector $2e^2m^*/\kappa\hbar^2$. For the electron concentration $n \sim 2 \times 10^{11}$ cm⁻², $\lambda_F \sim 600$ Å and the average thickness is $\Delta \sim 100$ Å. Therefore, we choose $q_{TF}/k_F = 2.0$ and $\Delta/\lambda_F = 0.2$.

Figure 1 shows the calculated coefficients $\tilde{C}$ and energy-gap width $|V_t|$ as a function of $k_F \hbar \omega_c / g$ in the case of $g/k_F = 1.2$ and constant strength of electrostatic modulation $V_0/E_F = 0.0125$. Solid lines are the results for $\delta = 0$ and dotted ones those for $\delta = \pi/2$. In the latter case, $|V_t|$ for $k_y > 0$ is different from that for $k_y < 0$ and the energy gaps for $k_y < 0$ close off at $k_g \hbar \omega_c / g = V_0$, i.e., $k_F \hbar \omega_c / g E_F \approx 0.016$ as described before.

The resistivity qualitatively reflects the energy-gap width as expected from the discussions in the previous

section though the curves show convexity due to divergence of the density of states [7]. The result for $\delta = \pi/2$ has a dip at $k_F \hbar \omega_c / g E_F \approx 0.016$ due to vanishing energy gaps for negative k_y and reduction of contribution of the dominant Umklapp processes. When the magnetic-field modulation becomes sufficiently large, $\tilde{C}$ for $\delta = 0$ and $\pi/2$ are similar to each other because energy gaps are determined mainly by the magnetic-field modulation.

Figure 2 shows some examples of the experimental results corresponding to $\tilde{C}(\hbar \omega_c) - \tilde{C}(0)$ in the case of $g/k_F = 0.12$ and $\delta = \pi/2$ [3]. The corresponding theoretical results are also shown. The theoretical result is in good agreement with the experiments although slightly overestimated.

There are ambiguities in experimental estimation of the absolute magnitude of the magnetic-field modulation because it depends heavily on the estimation of the strength of electrostatic modulation based on the Weiss oscillation [3]. There may also be ambiguities in T^2 fitting of the resistivity. Further, the calculation gives only an upper limit of the resistivity and therefore the resistivity is likely to be overestimated.

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Figure Captions

Fig. 1. Calculated $\tilde{C}$ and energy-gap width $|V_t|$ as a function of $k_F \hbar \omega_c / g$ for $V_0/E_F = 0.0125$.

Fig. 2. Calculated $\tilde{C}$ as a function of $k_F \hbar \omega_c / g$ in the realistic case. Filled circles are the results of the experiment [3].

Fig. 1

Fig. 2