エフィモフ効果と普遍性

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2021年12月20-22日

集中講義＠東北大学

Plan of this talk

1. Universality in physics

2. What is the Efimov effect ?

   Keywords: universality
   scale invariance
   quantum anomaly
   RG limit cycle

3. Beyond cold atoms: Quantum magnets

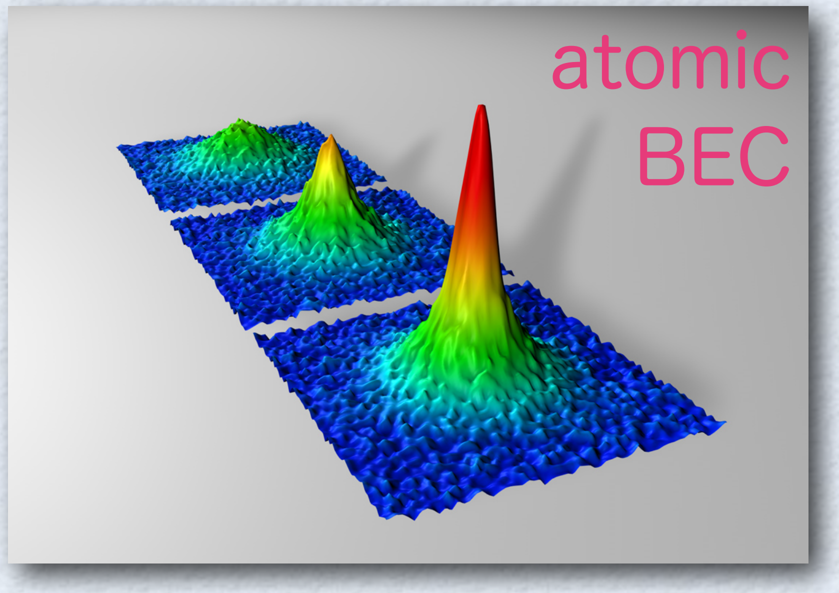
4. Recent progress: Super Efimov effect

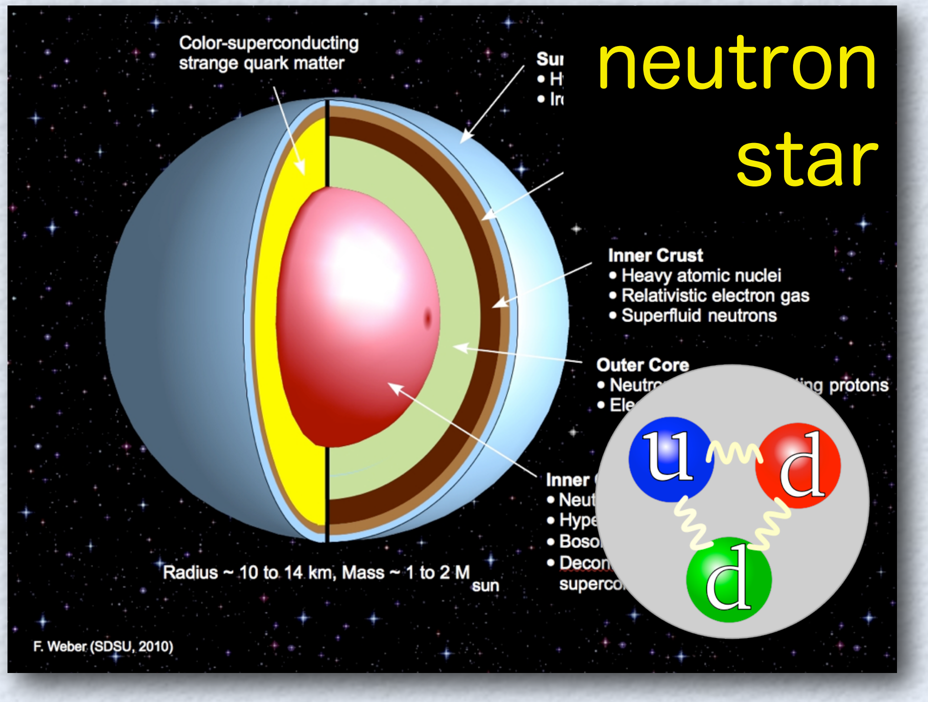
Introduction

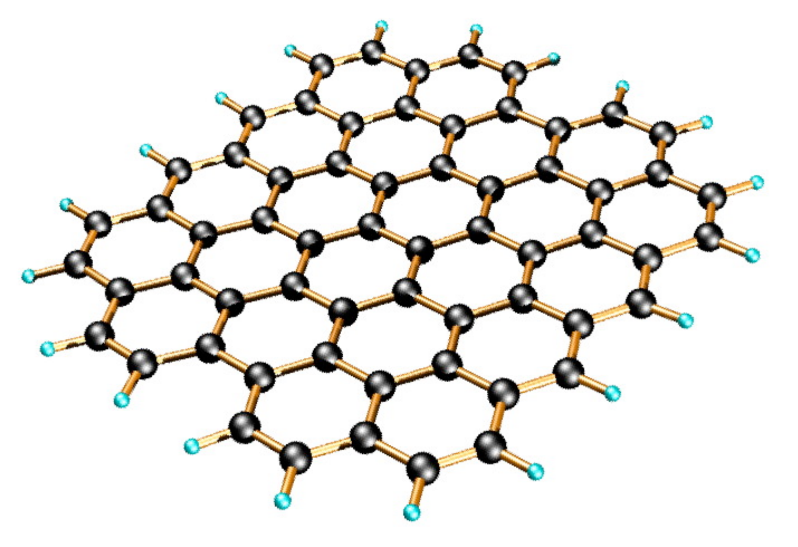
1. **Universality in physics**
2. What is the Efimov effect ?
3. Beyond cold atoms: Quantum magnets
4. Recent progress: Super Efimov effect

(ultimate) Goal of research

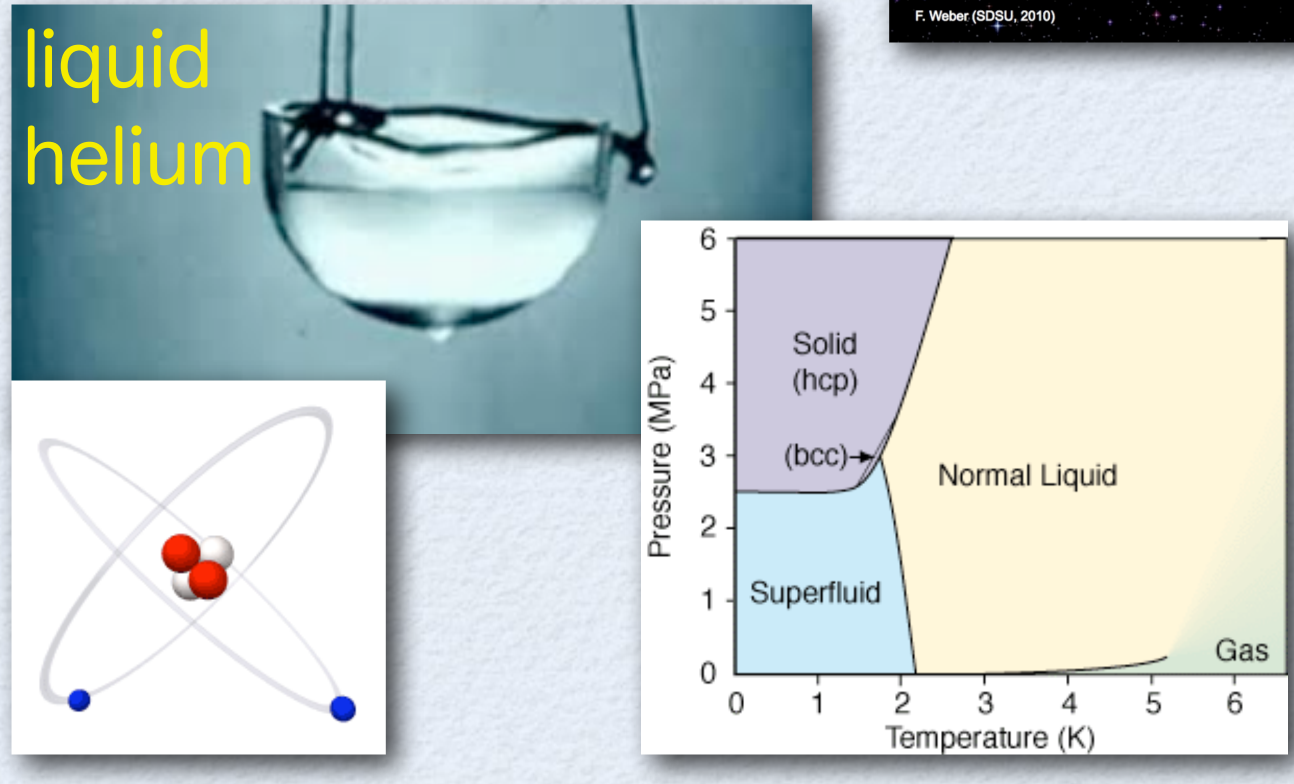
Understand physics of few and many particles governed by quantum mechanics


atomic BEC

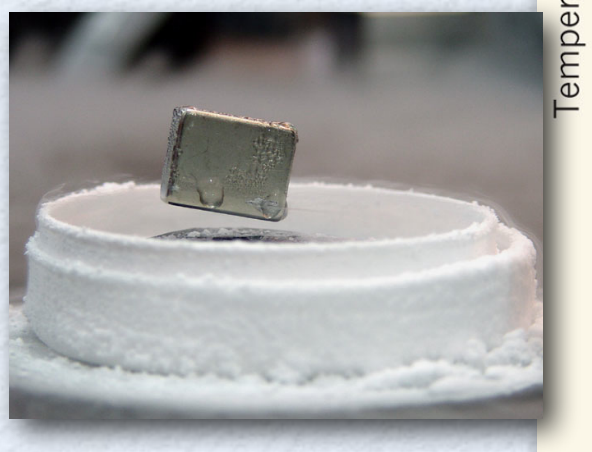

neutron star

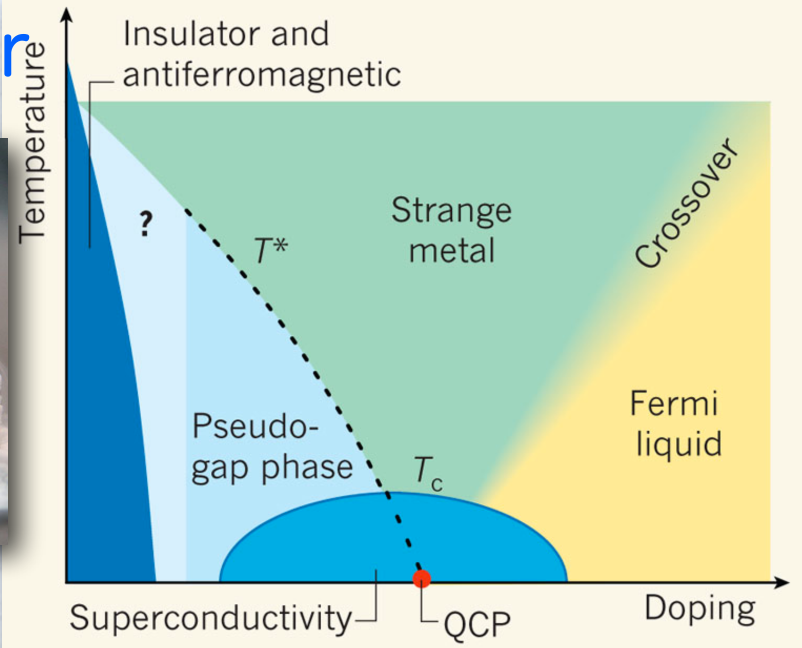

graphene

liquid helium



superconductor



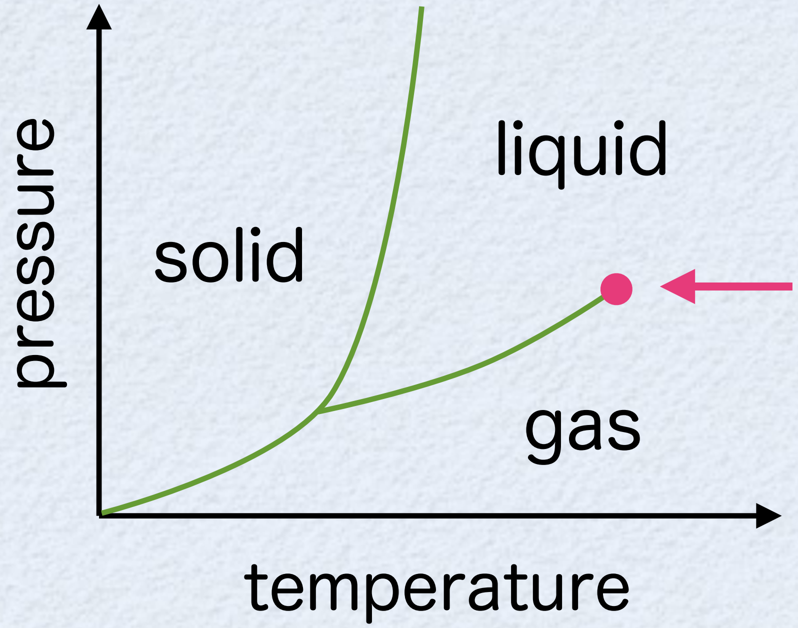


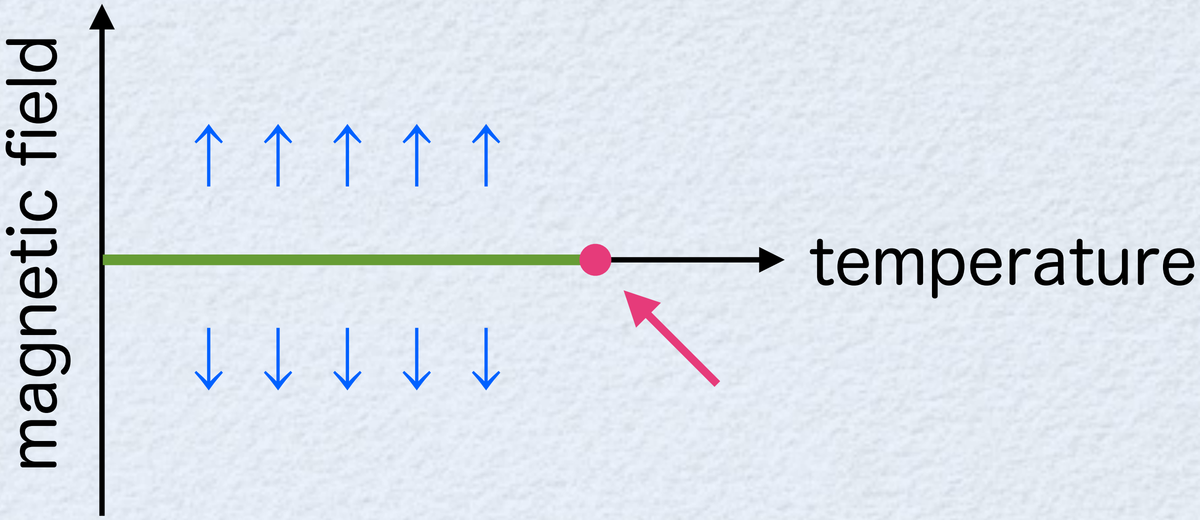
When physics is universal ?

A1. Continuous phase transitions ⇔ $\xi / r_0 \to \infty$

E.g. Water

vs. Magnet



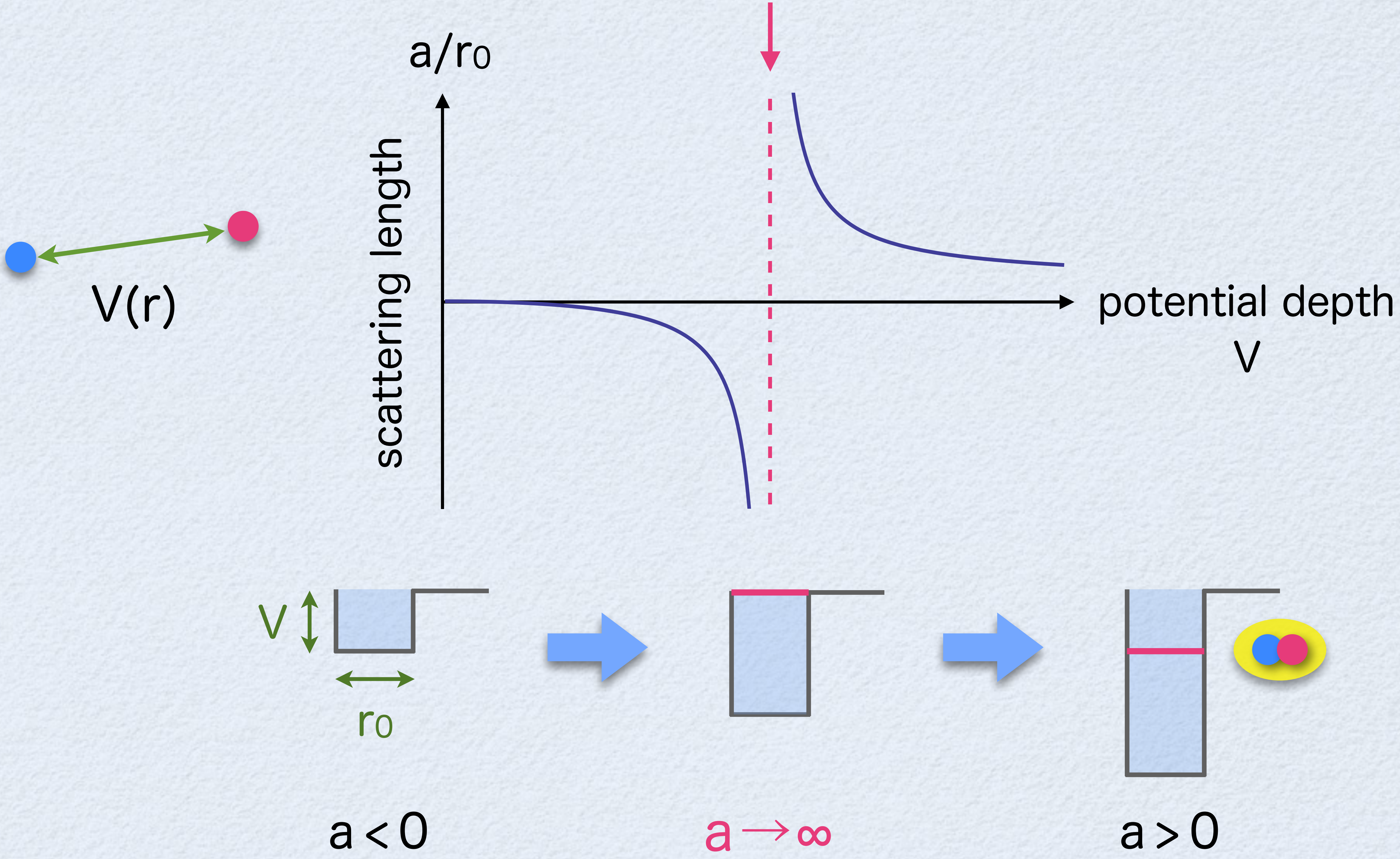


Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\mathrm{liq}} - \rho_{\mathrm{gas}} \sim (T_{\mathrm{c}} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\mathrm{c}} - T)^{\beta}$$

When physics is universal ?

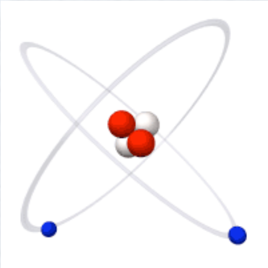
A2. Scattering resonances ⇔ $a/r_0 \to \infty$



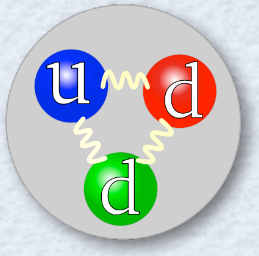
When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$

E.g. ^{4}He atoms

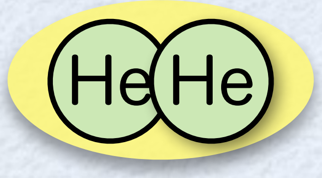


vs. proton/neutron



van der Waals force:

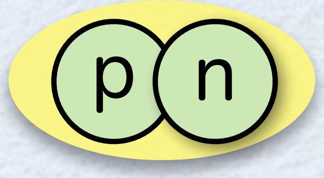
$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$



$E_{\text{binding}} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{\text{binding}} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\text{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

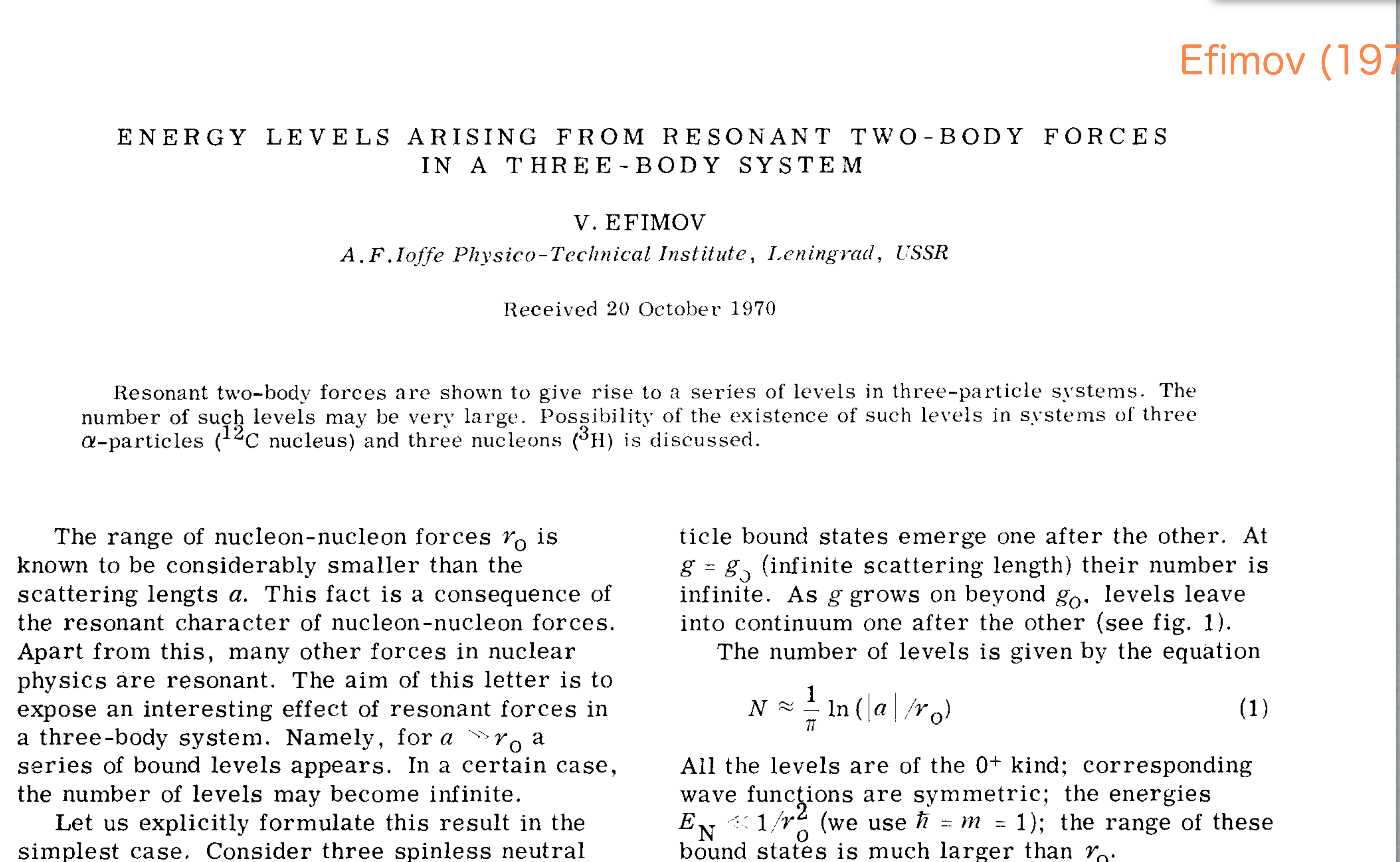
➡ Physics only depends on the scattering length "a"

Efimov effect

1. Universality in physics
2. **What is the Efimov effect ?**
3. Beyond cold atoms: Quantum magnets
4. Recent progress: Super Efimov effect

Efimov effect

Efimov (1970)



Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^3H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

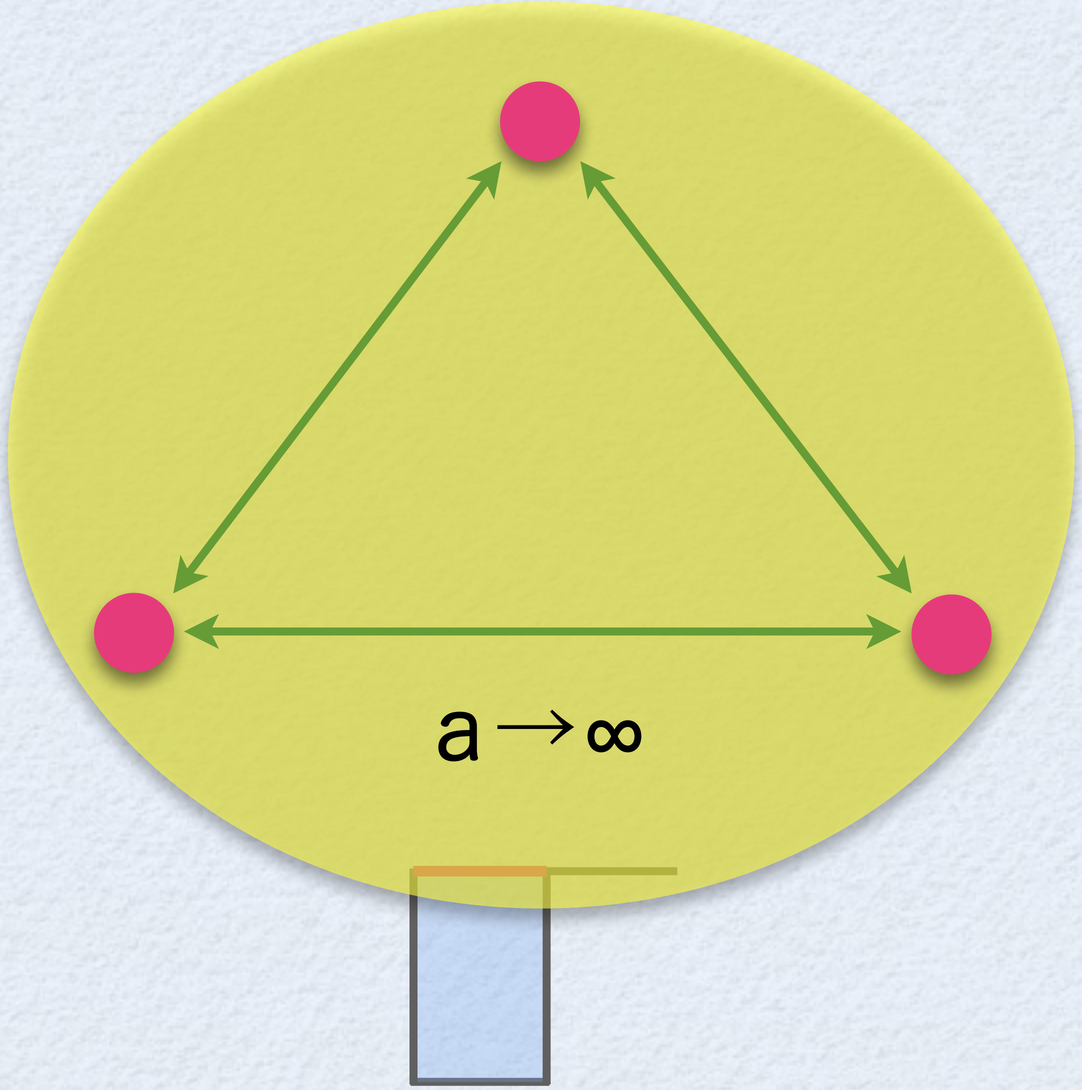
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



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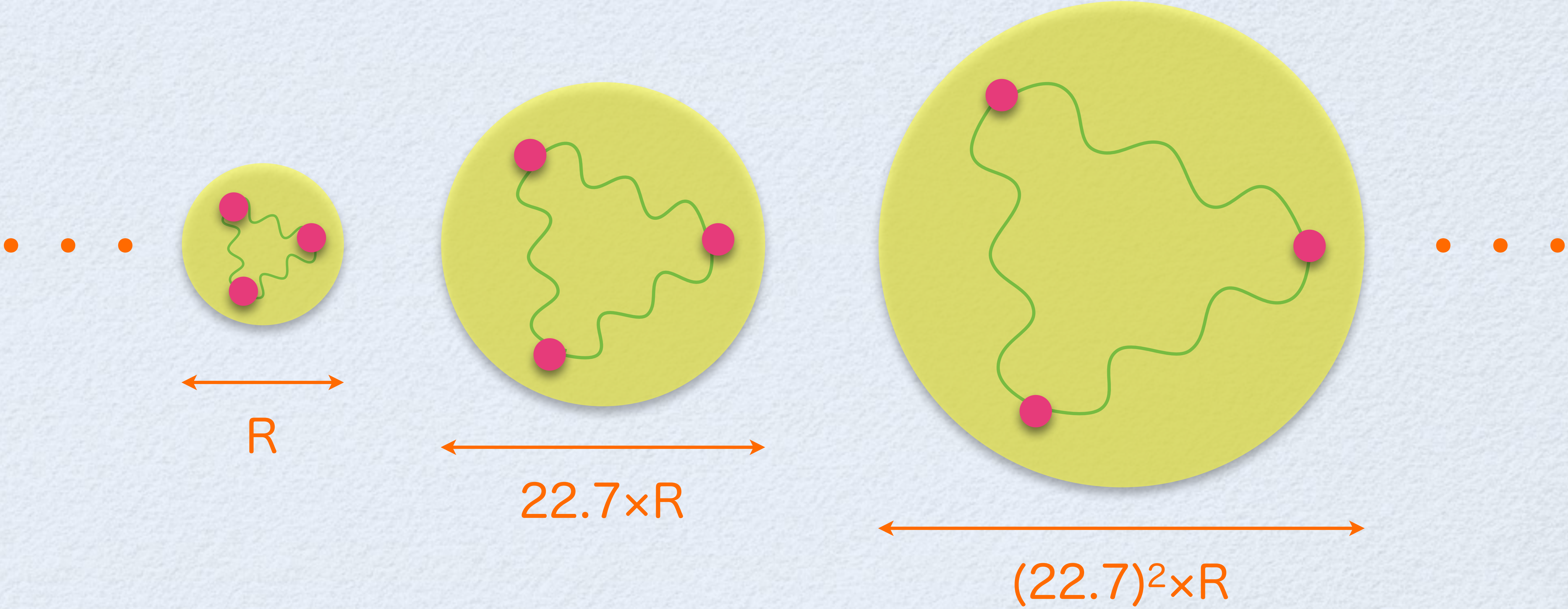


Discrete scaling symmetry

Why Efimov effect happens ?

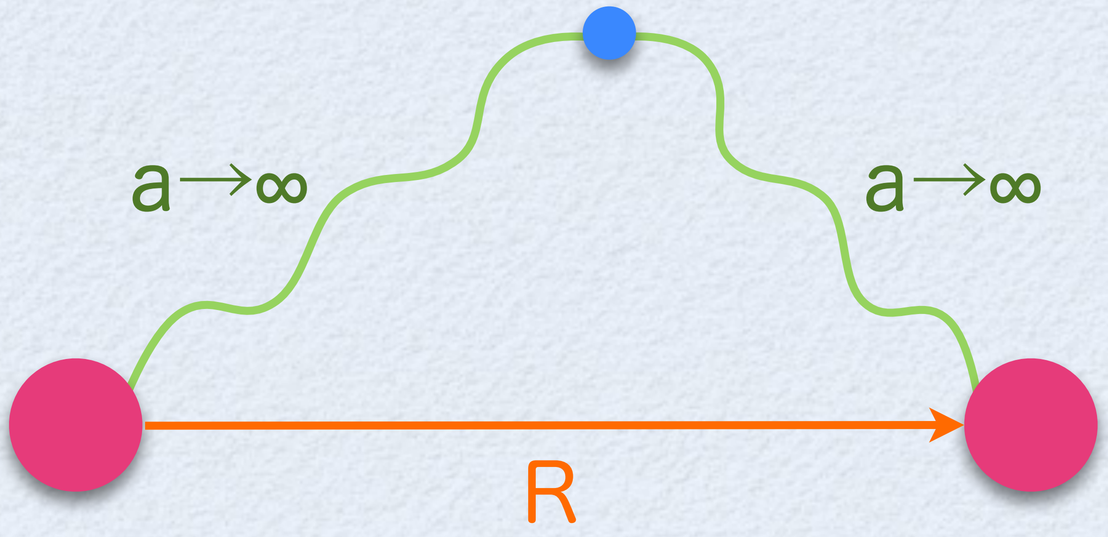
Keywords

✓ Universality
- Scale invariance
- Quantum anomaly
- RG limit cycle

Why Efimov effect happens ?

Two heavy (M) and one light (m) particles

➡ Born-Oppenheimer approximation



Binding energy of a light particle

$$E_b(R) = -\boxed{\frac{\hbar^2}{2mR^2}} \times (0.5671\ldots)^2$$

Scale invariance at a→∞

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial \boldsymbol{R}^2} + V(R)\right]\psi(\boldsymbol{R}) = -\frac{\hbar^2\kappa^2}{M}\psi(\boldsymbol{R}) \qquad V(R) \equiv E_b(R)$$

Why Efimov effect happens ?

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\left(\frac{\partial^2}{\partial R^2}+\frac{2}{R}\frac{\partial}{\partial R}\right)-\frac{\hbar^2}{2mR^2}(0.5671\ldots)^2\right]\psi(R)=-\frac{\hbar^2\kappa^2}{M}\psi(R)$$

$$\psi(R)=R^{-1/2}K_{i\alpha}(\kappa R) \qquad \alpha^2\equiv\frac{M}{2m}(0.5671\ldots)^2-\frac{1}{4}$$

$$\to R^{-1/2}\sin[\alpha\ln(\kappa R)+\delta] \qquad (R\to 0)$$

ψ'/ψ has to be fixed by short-range physics

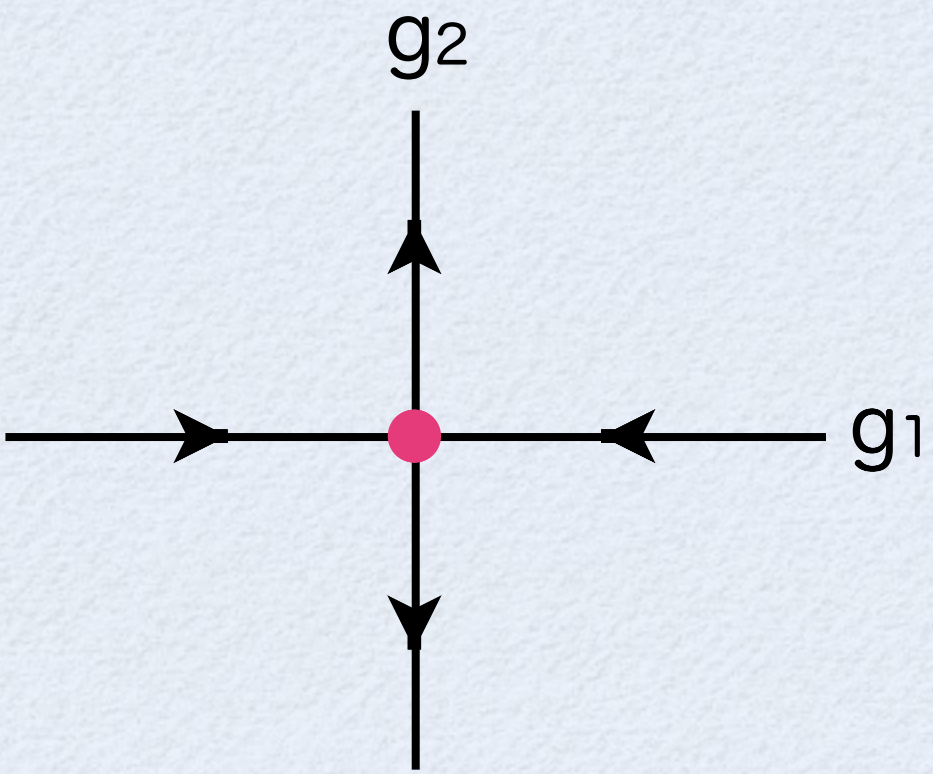
➡ If $\kappa=\kappa_*$ is a solution, $\kappa=(e^{\pi/\alpha})^n\kappa_*$ are solutions!

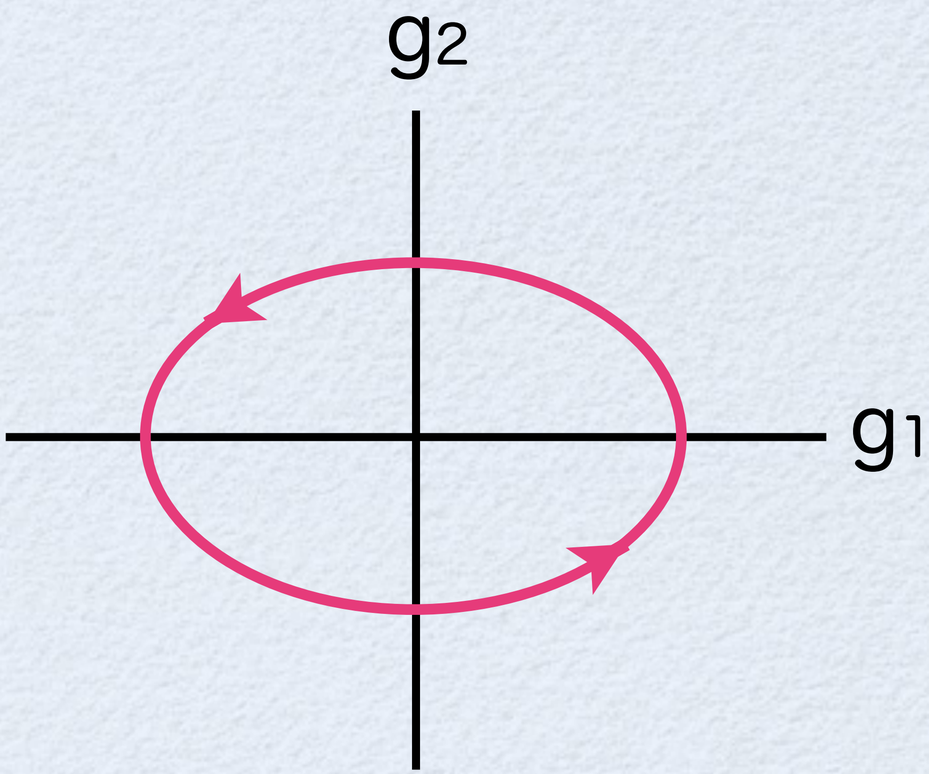
Classical scale invariance is broken by κ_*

= Quantum anomaly

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

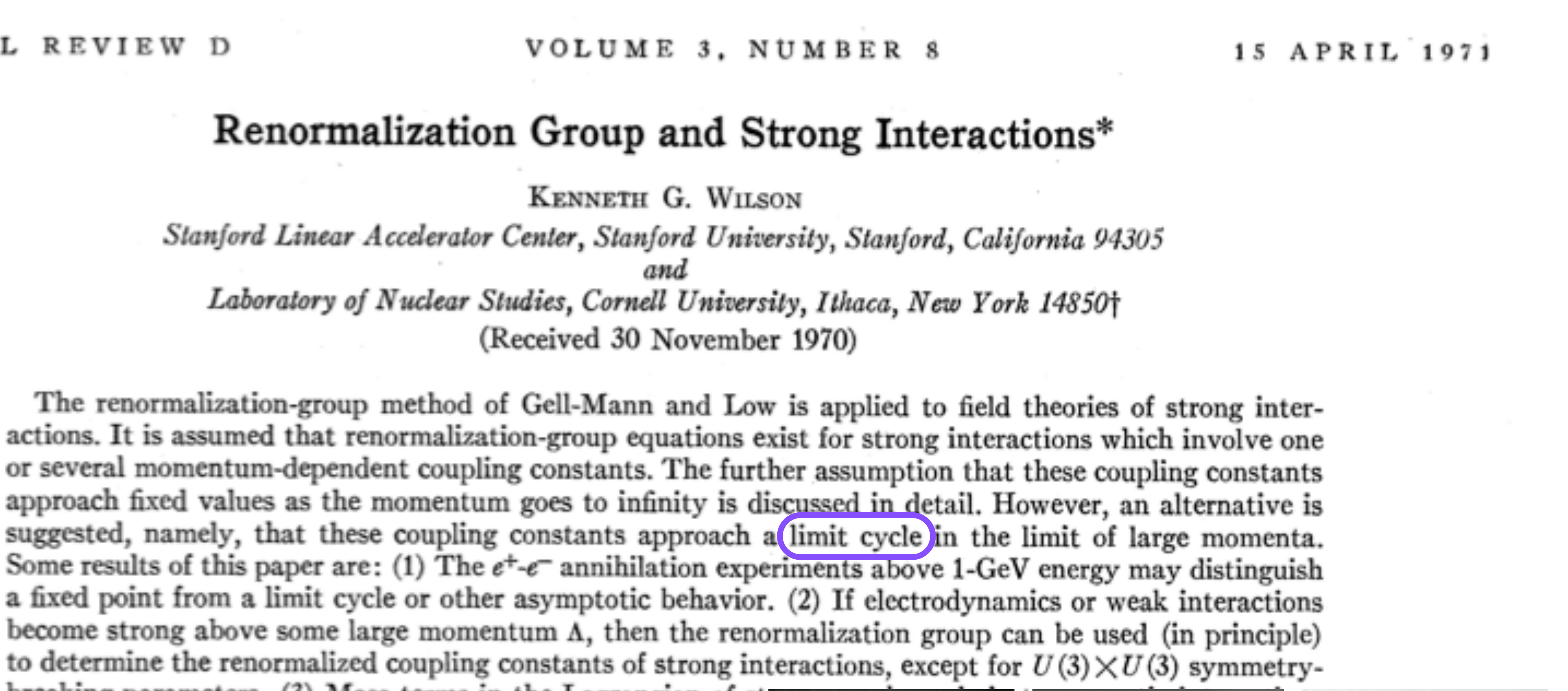
RG limit cycle

⇒ Discrete scale invariance

E.g. E????v effect

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions



L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

Kenneth G. Wilson

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†

(Received 30 November 1970)

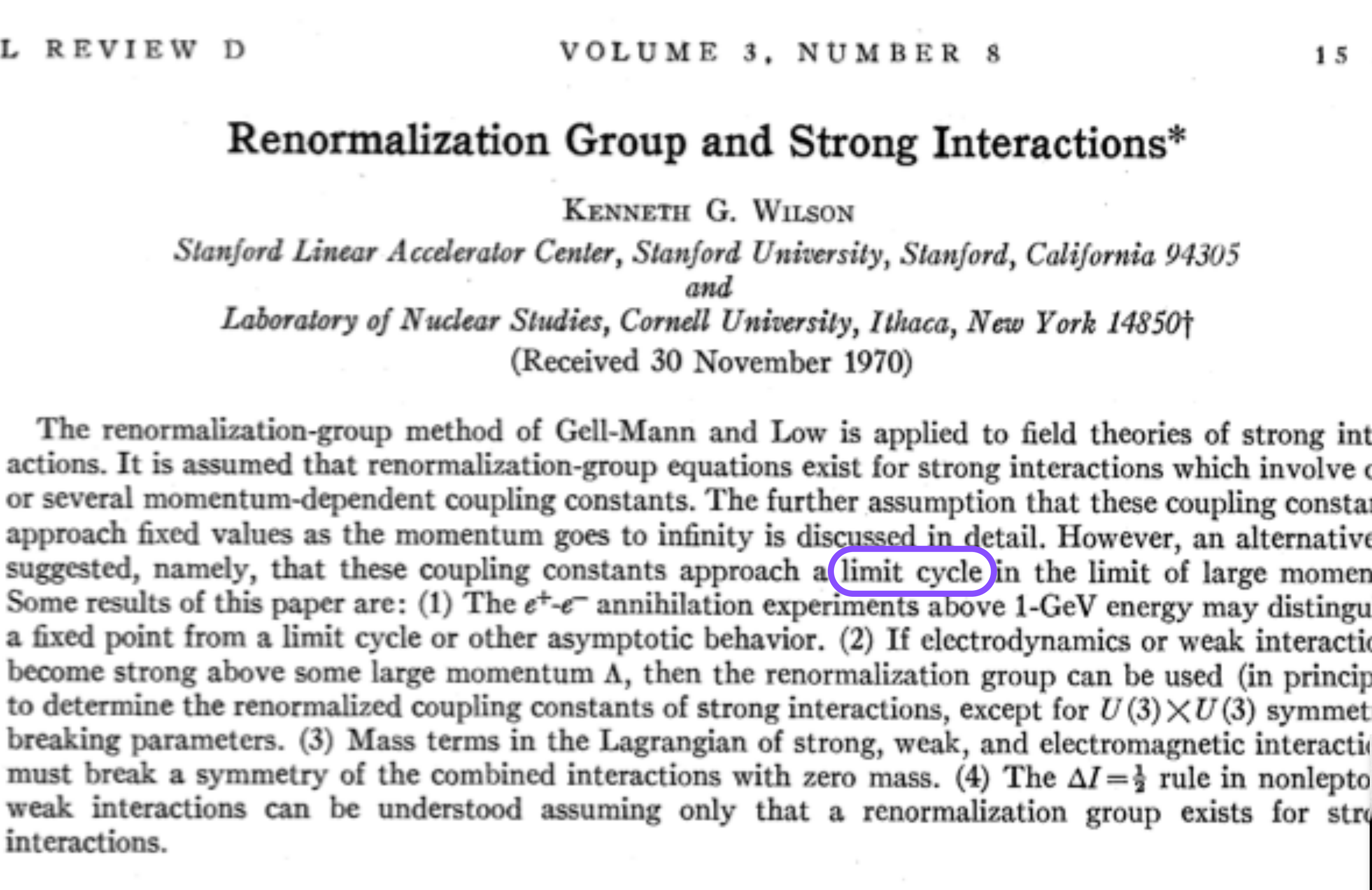
The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of st
must break a symmetry of the combined interactions with z
weak interactions can be understood assuming only that
interactions.

QCD is asymptotic free

(2004 Nobel prize)

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions



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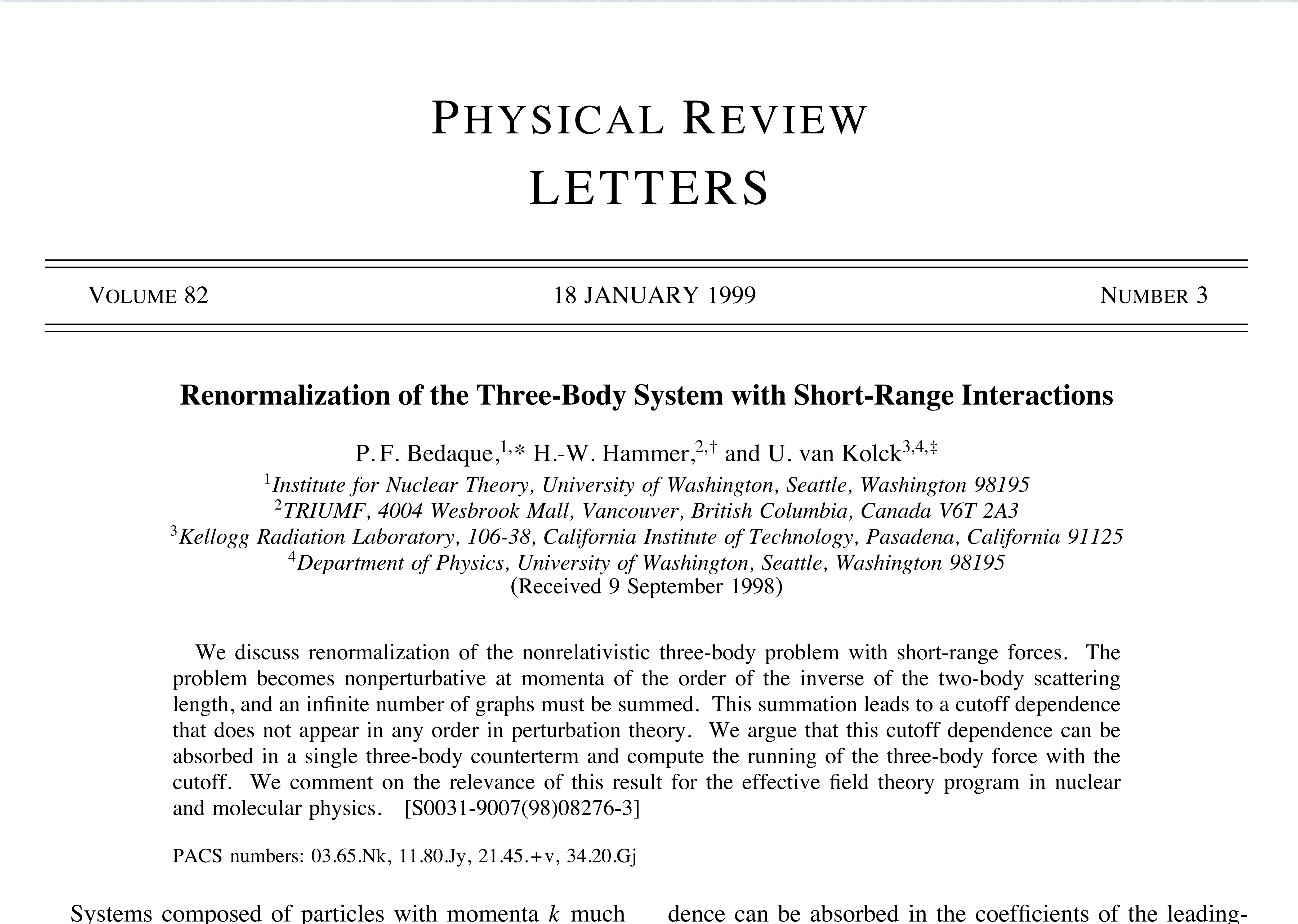
Efimov effect (1970) is its rare manifestation!

Effective field theory



PHYSICAL REVIEW
LETTERS

VOLUME 82 18 JANUARY 1999 NUMBER 3

Renormalization of the Three-Body System with Short-Range Interactions

P.F. Bedaque,[1,*] H.-W. Hammer,[2,†] and U. van Kolck[3,4,‡]
[1] *Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195*
[2] *TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*
[3] *Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125*
[4] *Department of Physics, University of Washington, Seattle, Washington 98195*
(Received 9 September 1998)

We discuss renormalization of the nonrelativistic three-body problem with short-range forces. The problem becomes nonperturbative at momenta of the order of the inverse of the two-body scattering length, and an infinite number of graphs must be summed. This summation leads to a cutoff dependence that does not appear in any order in perturbation theory. We argue that this cutoff dependence can be absorbed in a single three-body counterterm and compute the running of the three-body force with the cutoff. We comment on the relevance of this result for the effective field theory program in nuclear and molecular physics. [S0031-9007(98)08276-3]

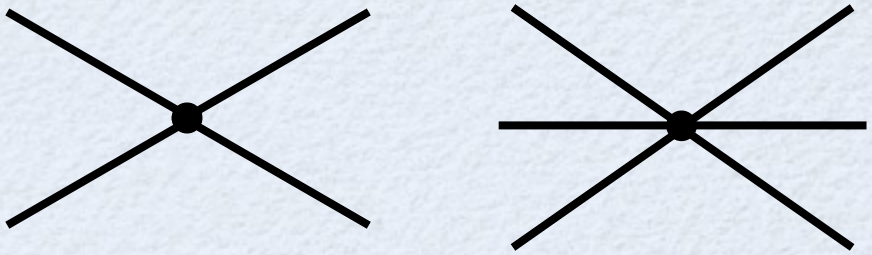
PACS numbers: 03.65.Nk, 11.80.Jy, 21.45.+v, 34.20.Gj

Systems composed of particles with momenta k much

dence can be absorbed in the coefficients of the leading-

Effective field theory

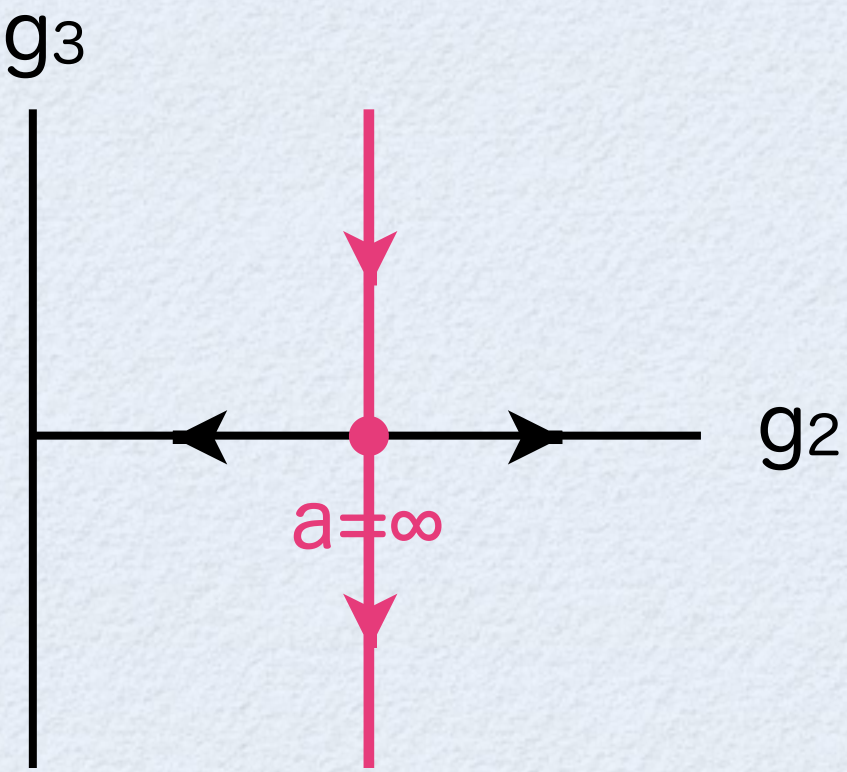
$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + g_2(\psi^\dagger\psi)^2 + g_3(\psi^\dagger\psi)^3$$



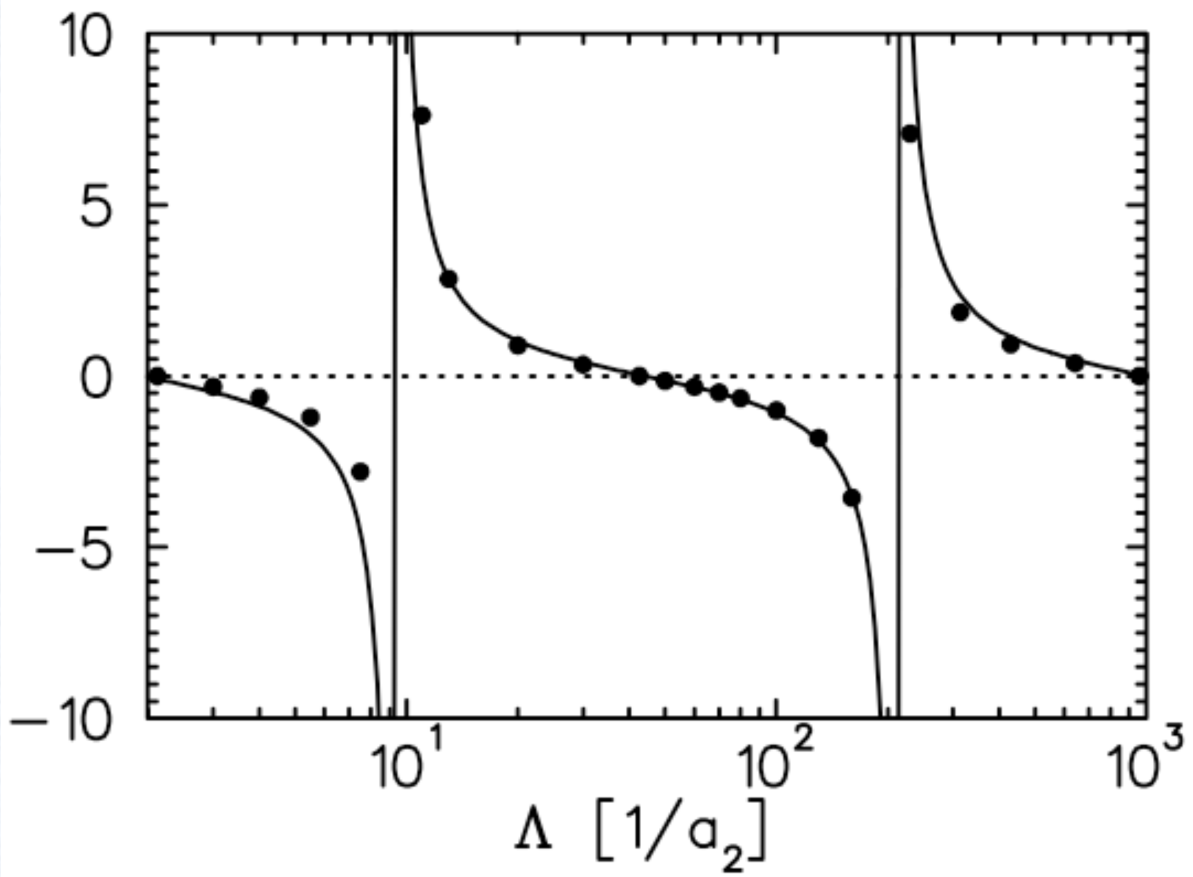
g_2 has a fixed point corresponding to a=∞

What is flow of g_3 ?

$$g_3(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$



RG limit cycle



Effective field theory

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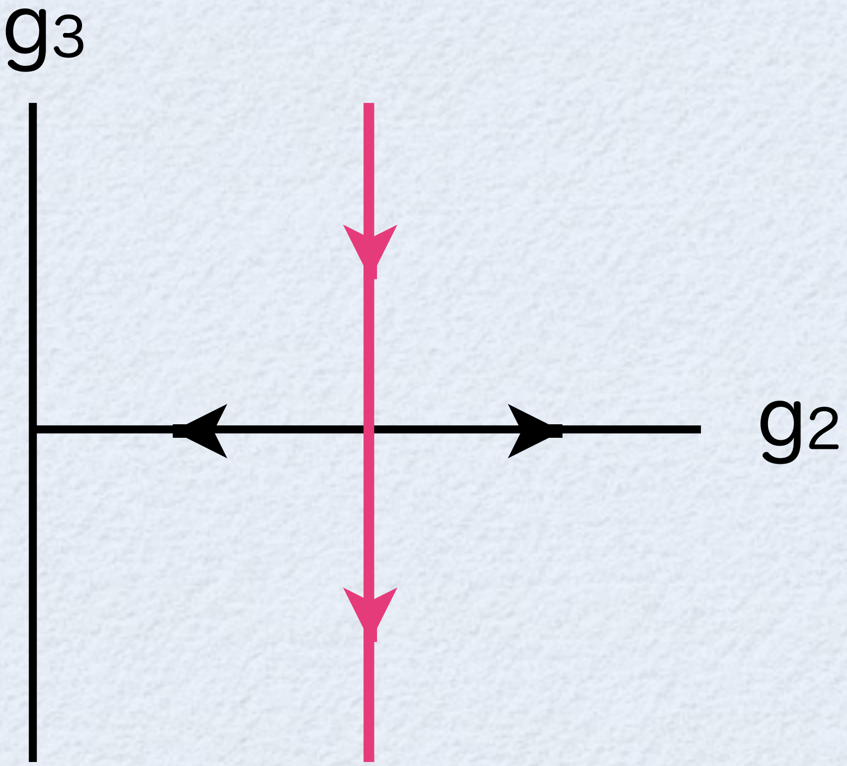
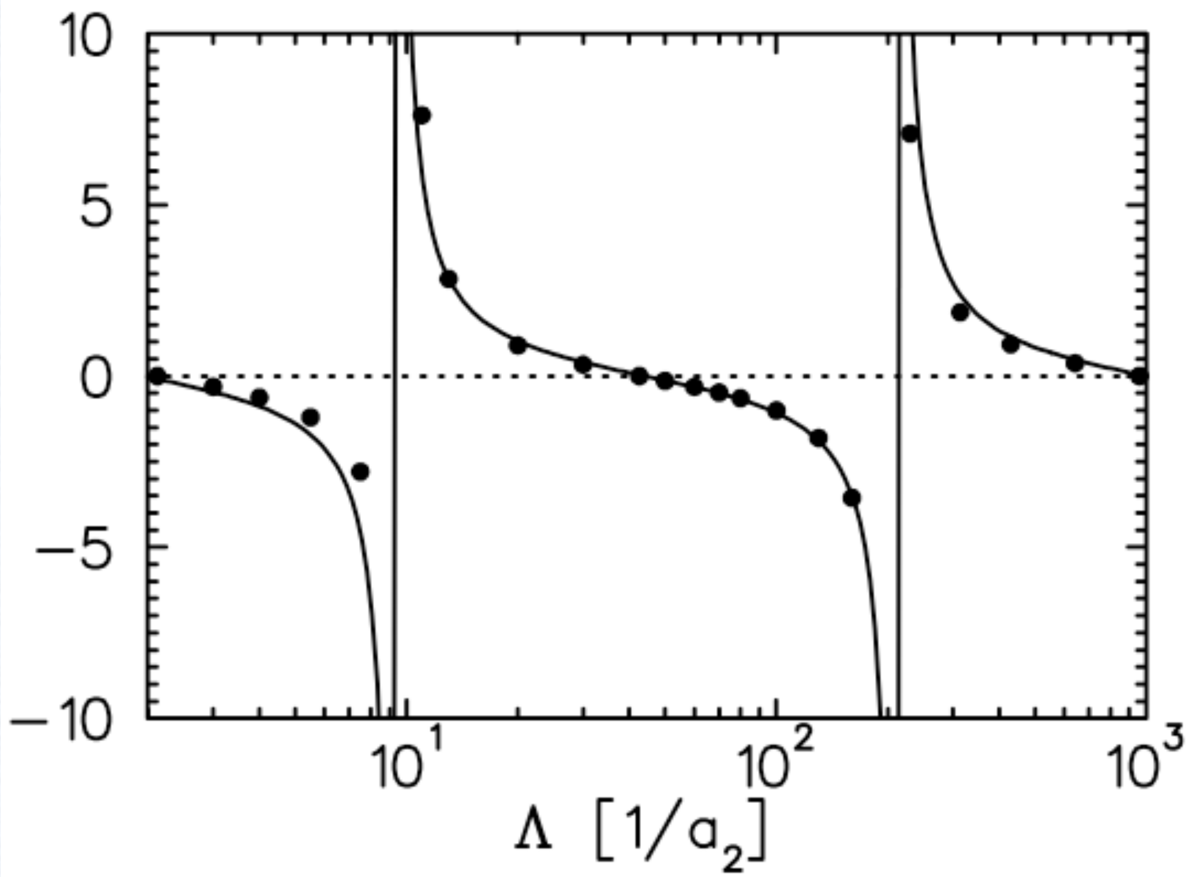
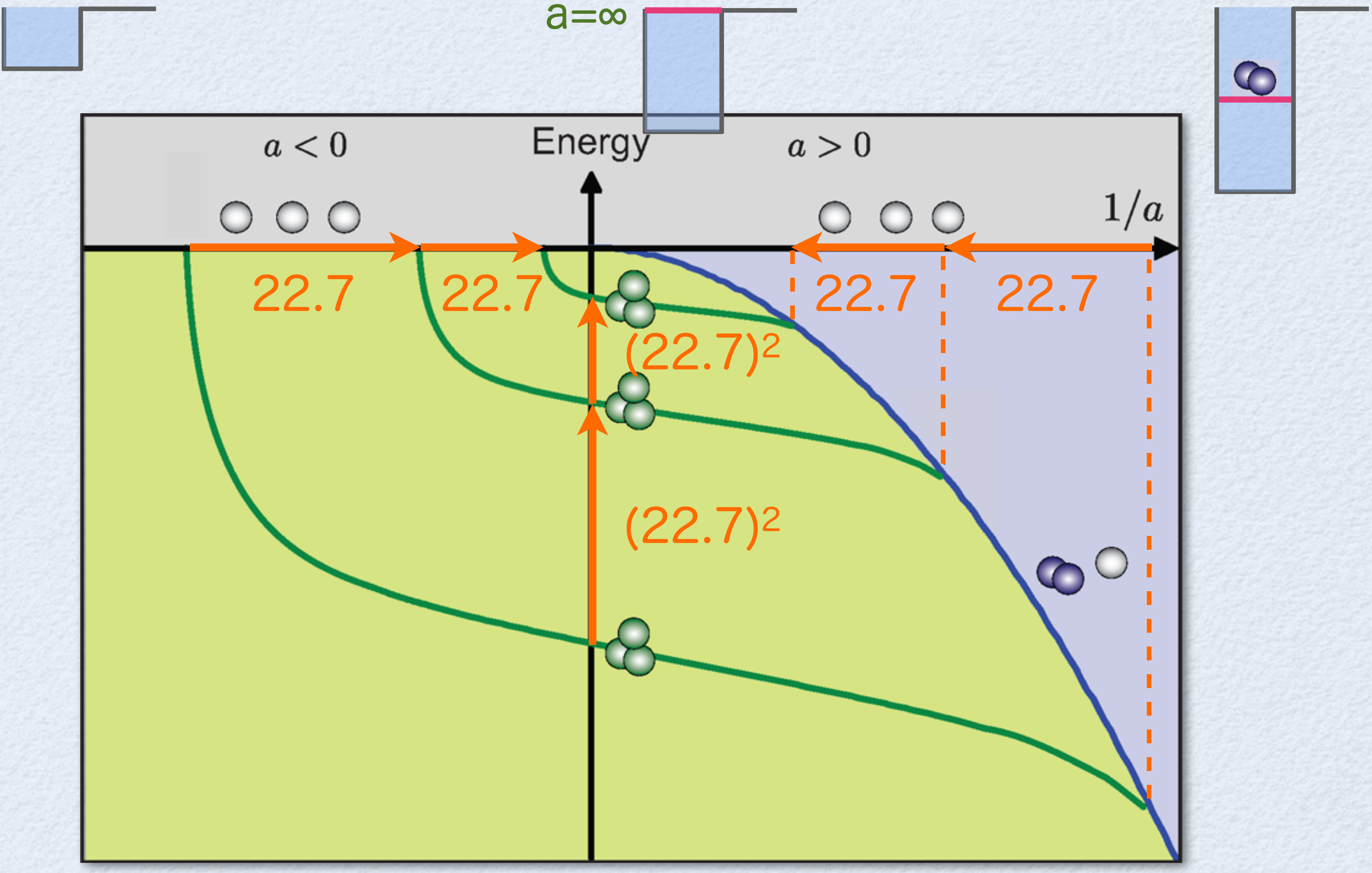


RG limit cycle



Efimov effect at a≠∞



Ferlaino & Grimm, Physics (2010)

Discrete scaling symmetry

Why 22.7 ?

Just a numerical number given by

22.694382595366695192860217 1369...

log(22.694382595366695192860217 1369...)

= 3.1221174311042196807309173 2438...

= π / 1.006237825102781489064066 81234...

= π / s_0

$$\frac{2\pi}{s_0}\,\frac{\sinh(\frac{\pi}{6}s_0)}{\cosh(\frac{\pi}{2}s_0)} = \frac{\sqrt{3}\pi}{4}$$

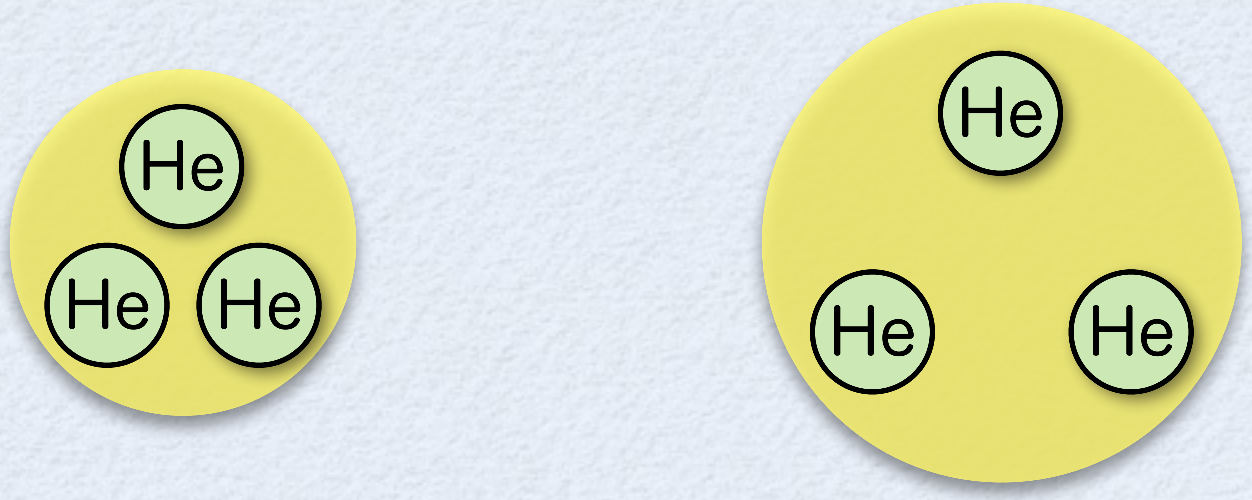
22.7 = exp(π / 1.006...)

Where Efimov effect appears ?

× Originally, Efimov considered
^{3}H nucleus (≈3 n) and ^{12}C nucleus (≈3 α)

△ ^{4}He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted
and observed in 1994 and 2015

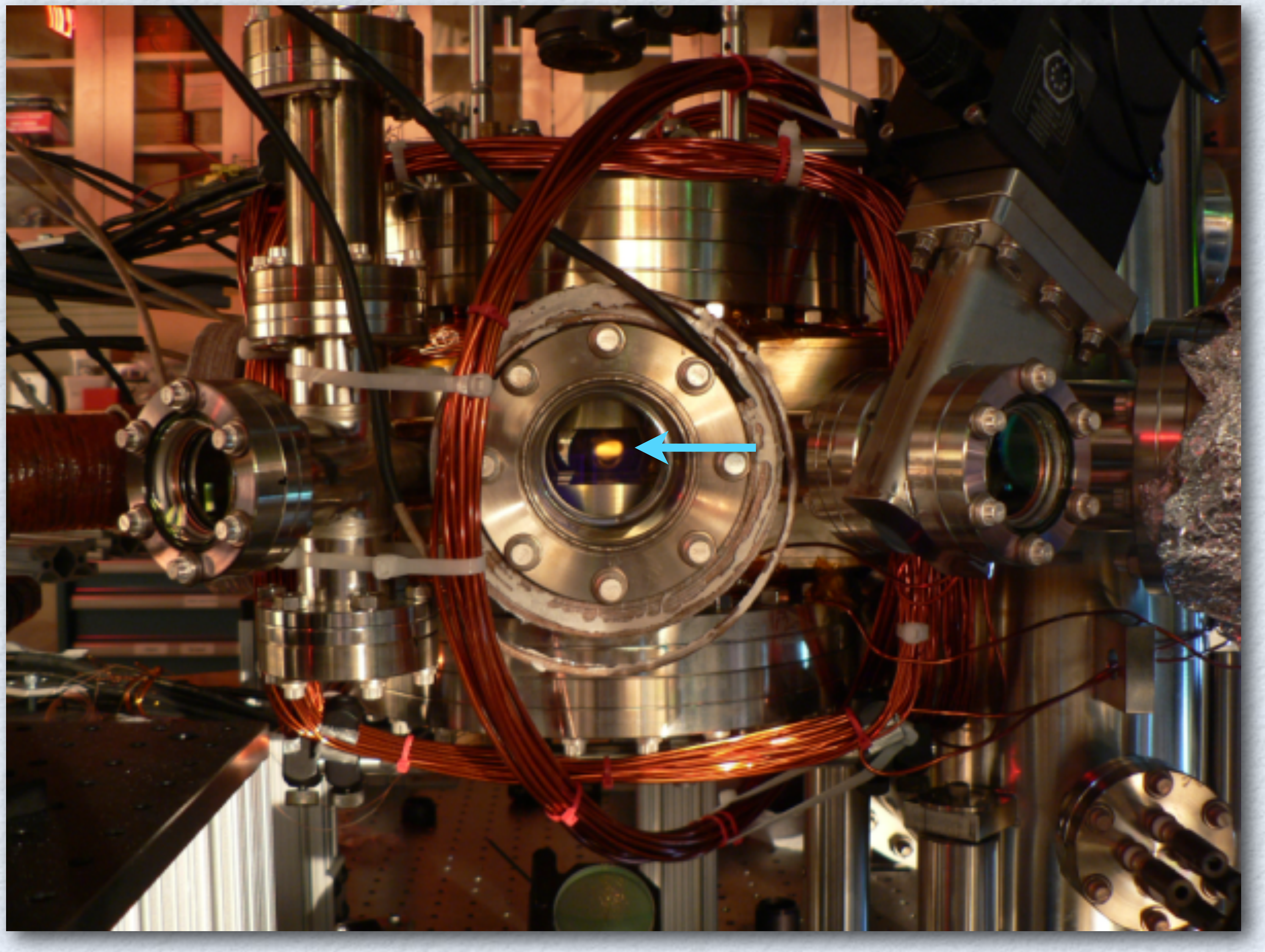


E_b = 125.8 mK E_b = 2.28 mK

but no discrete scaling → Ultracold atoms !

Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will



Ultracold atom experiments

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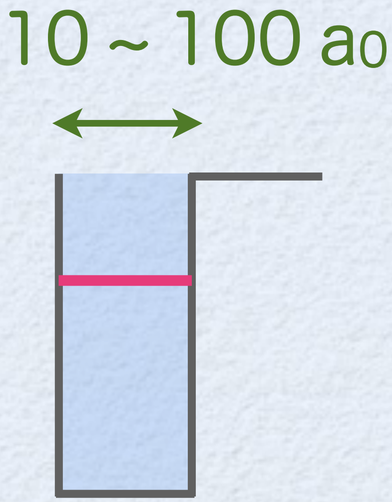
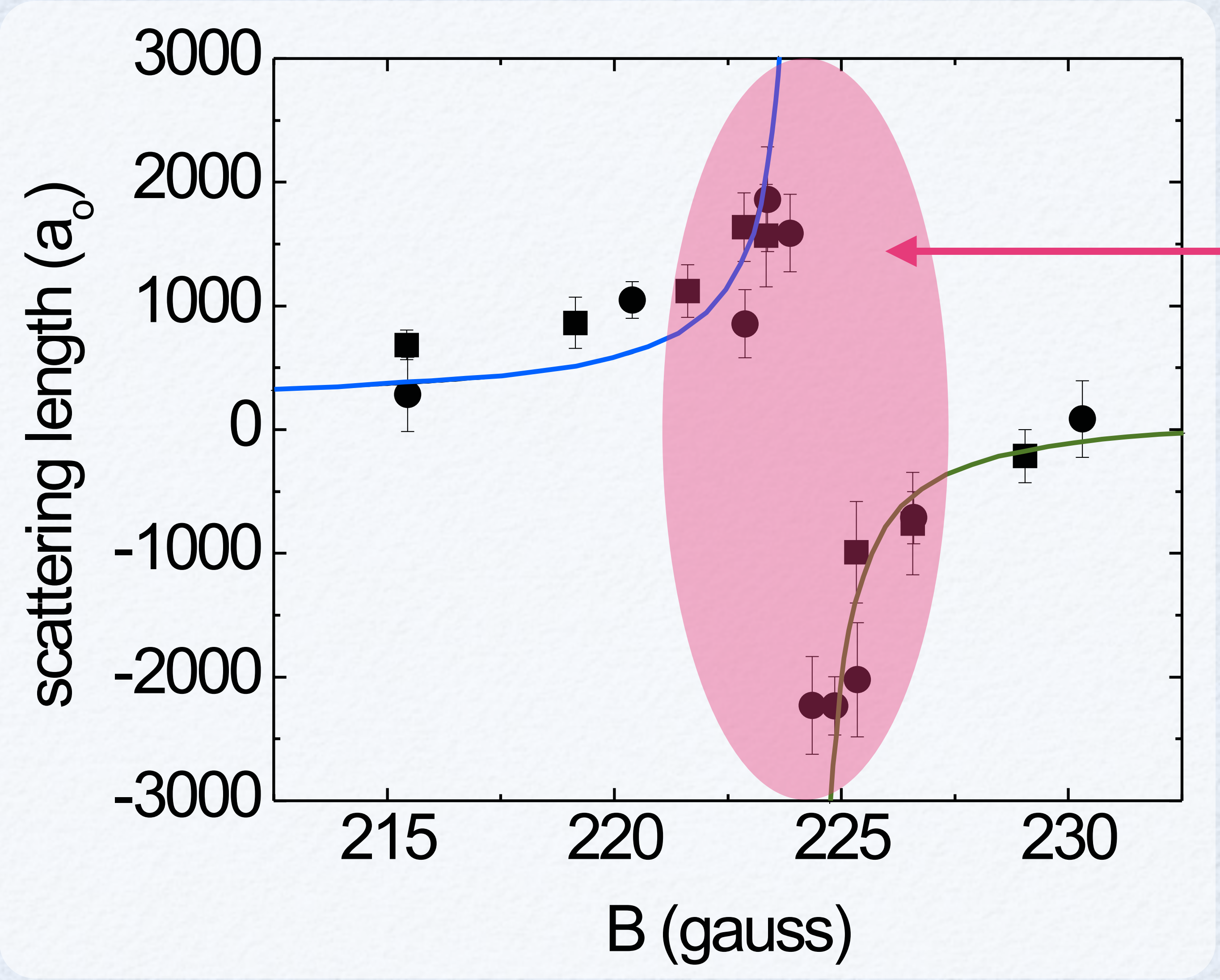
✓ Interaction strength by Feshbach resonances





Universal regime

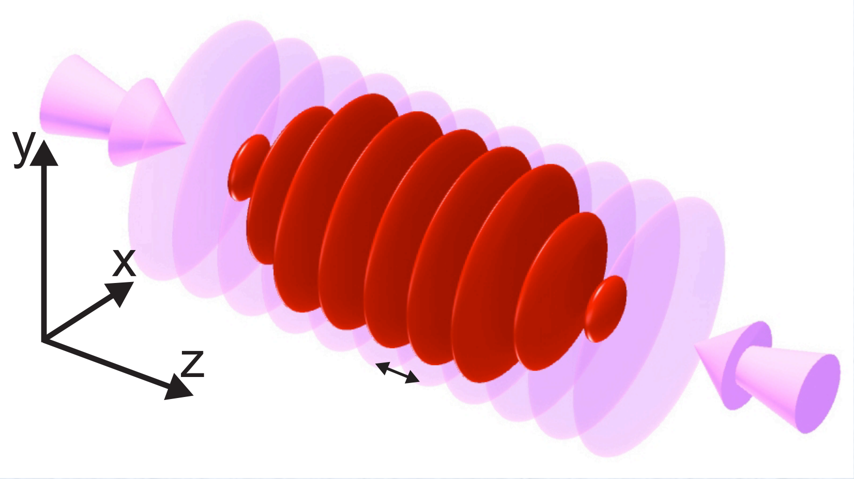
C.A. Regal & D.S. Jin
PRL90 (2003)

Ultracold atom experiments

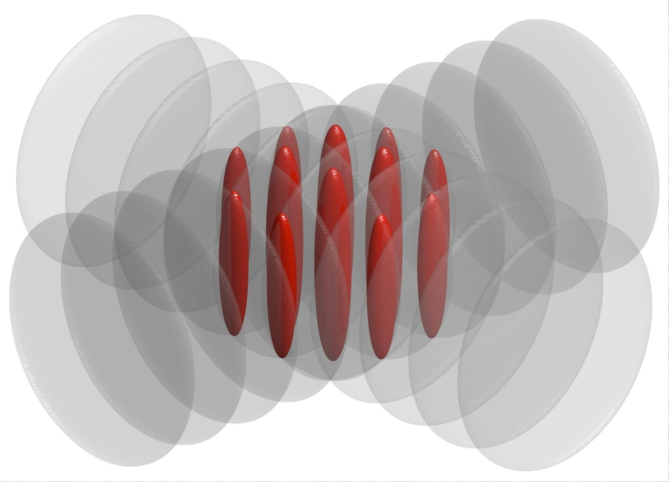
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

- ✓ Interaction strength by Feshbach resonances
- ✓ Spatial dimensions by strong optical lattices

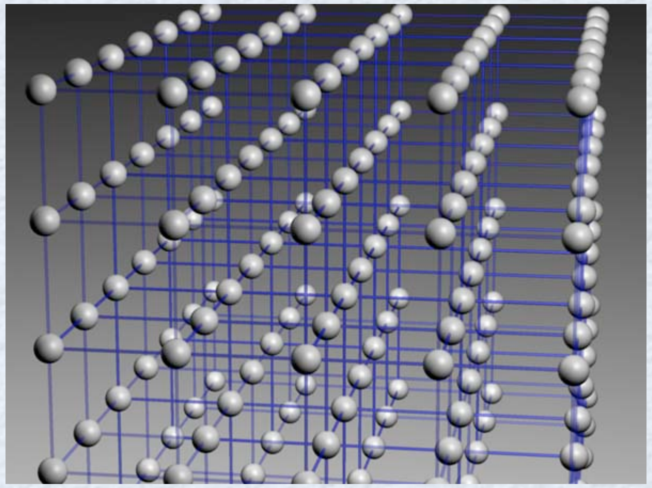
2D

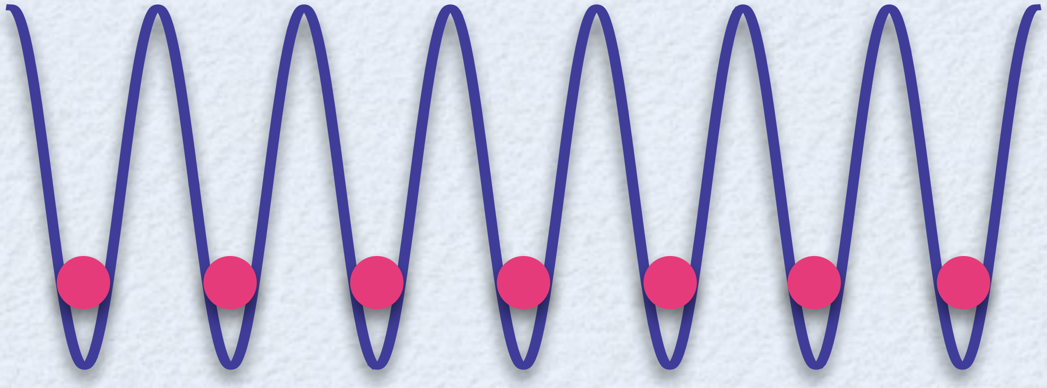


1D



0D





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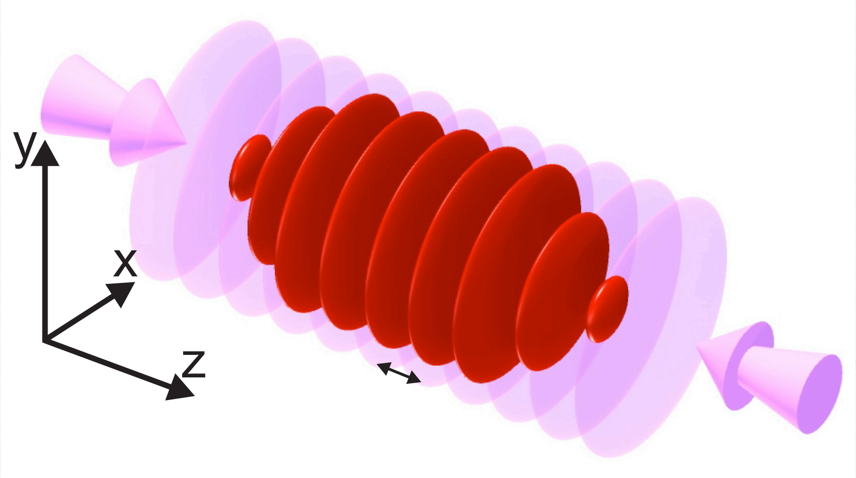
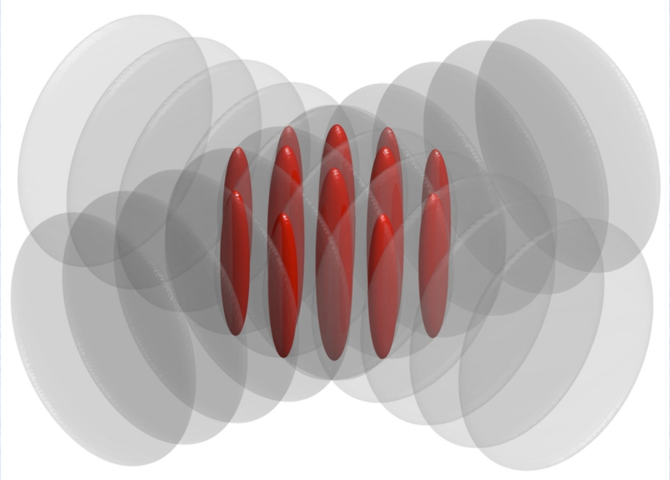
- ✓ Interaction strength by Feshbach resonances
- ✓ Spatial dimensions by strong optical lattices

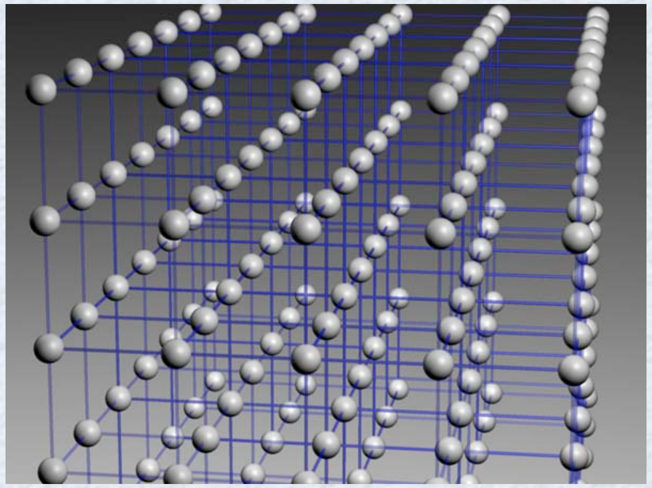
2D



1D



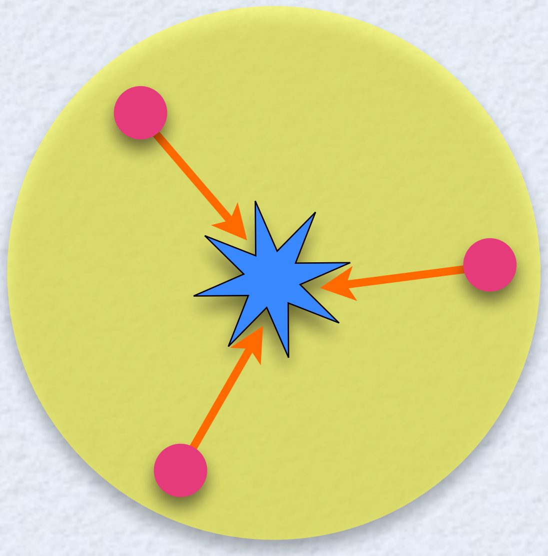
0D



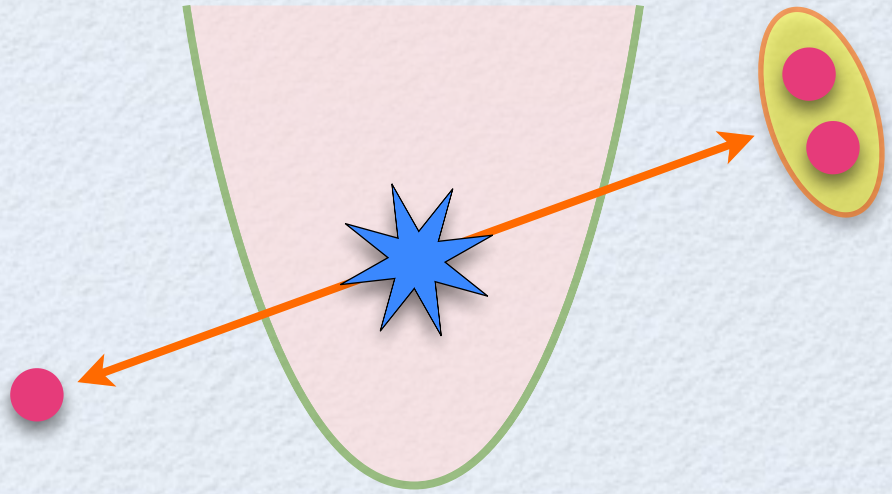
- ✓ Quantum statistics of particles
  - Bosonic atoms (^{7}Li, ^{23}Na, ^{39}K, ^{41}K, ^{87}Rb, ^{133}Cs, ...)
  - Fermionic atoms (^{6}Li, ^{40}K, ...)

Ultracold atom experiments

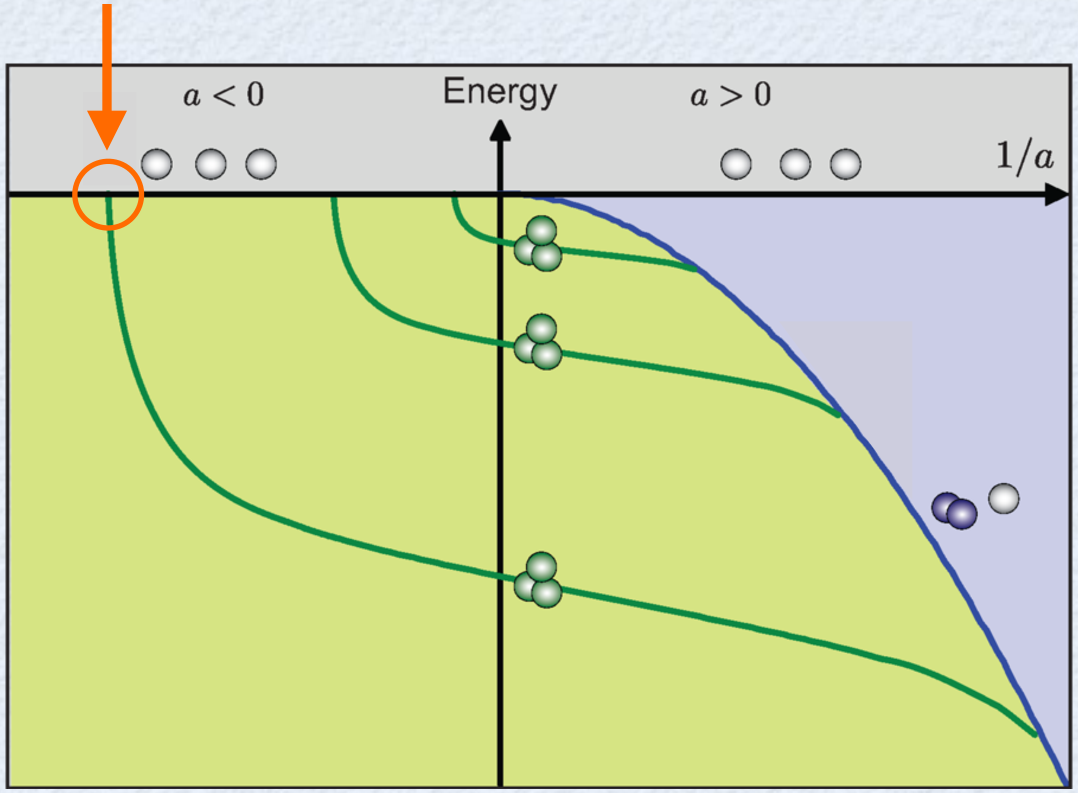
First experiment by Innsbruck group for ^{133}Cs (2006)

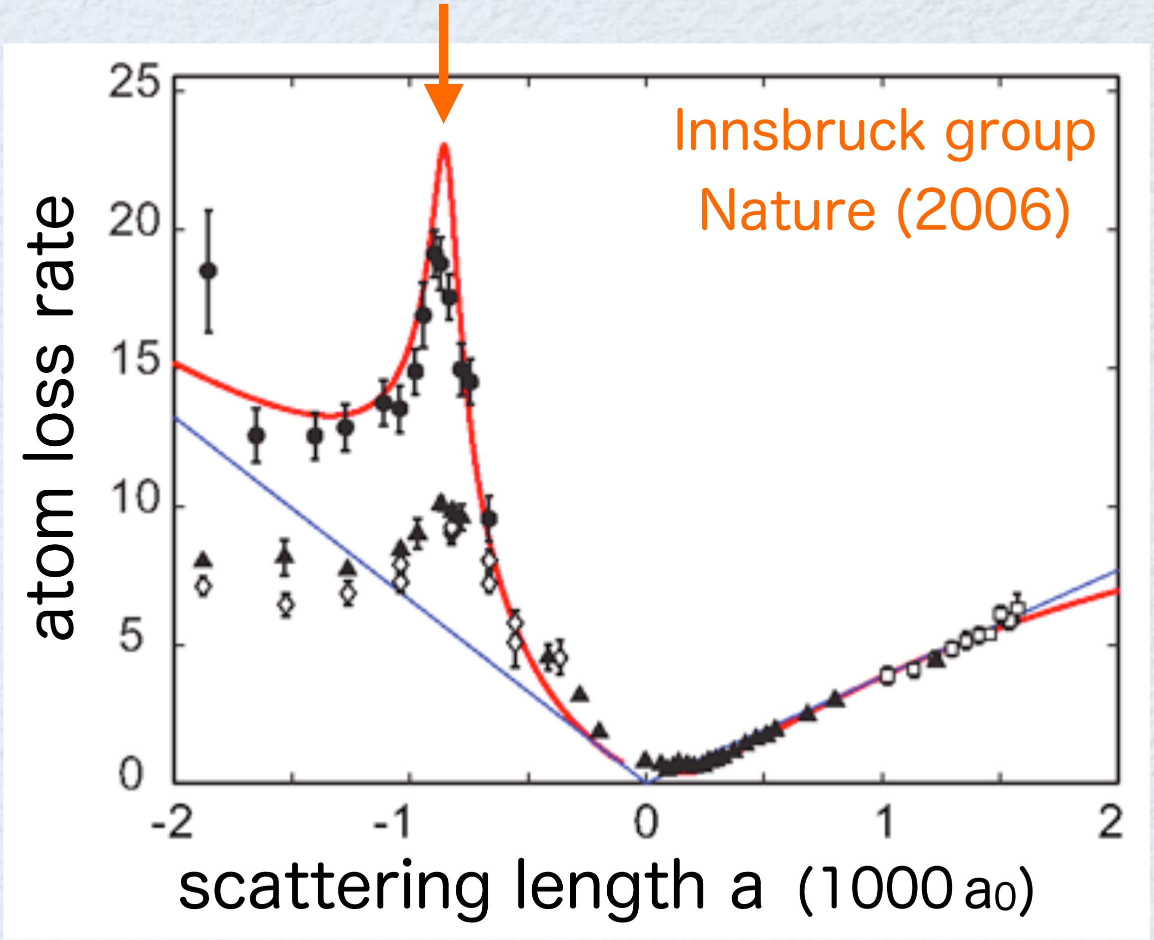


Trimer is unstable → atom loss



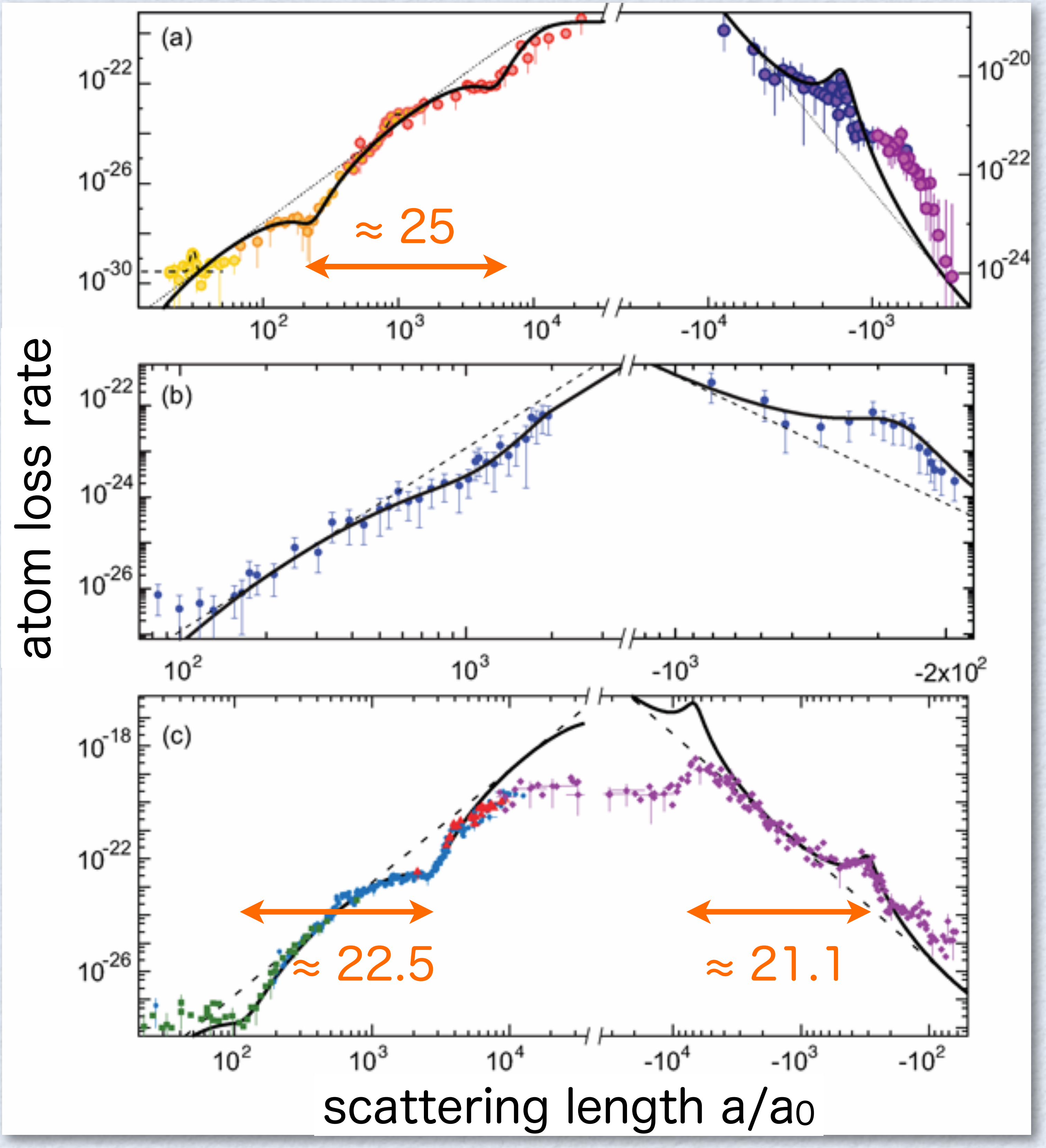
signature of trimer formation





Innsbruck group
Nature (2006)

Ultracold atom experiments



Florence group for ^{39}K (2009)

Bar-Ilan University for ^{7}Li (2009)

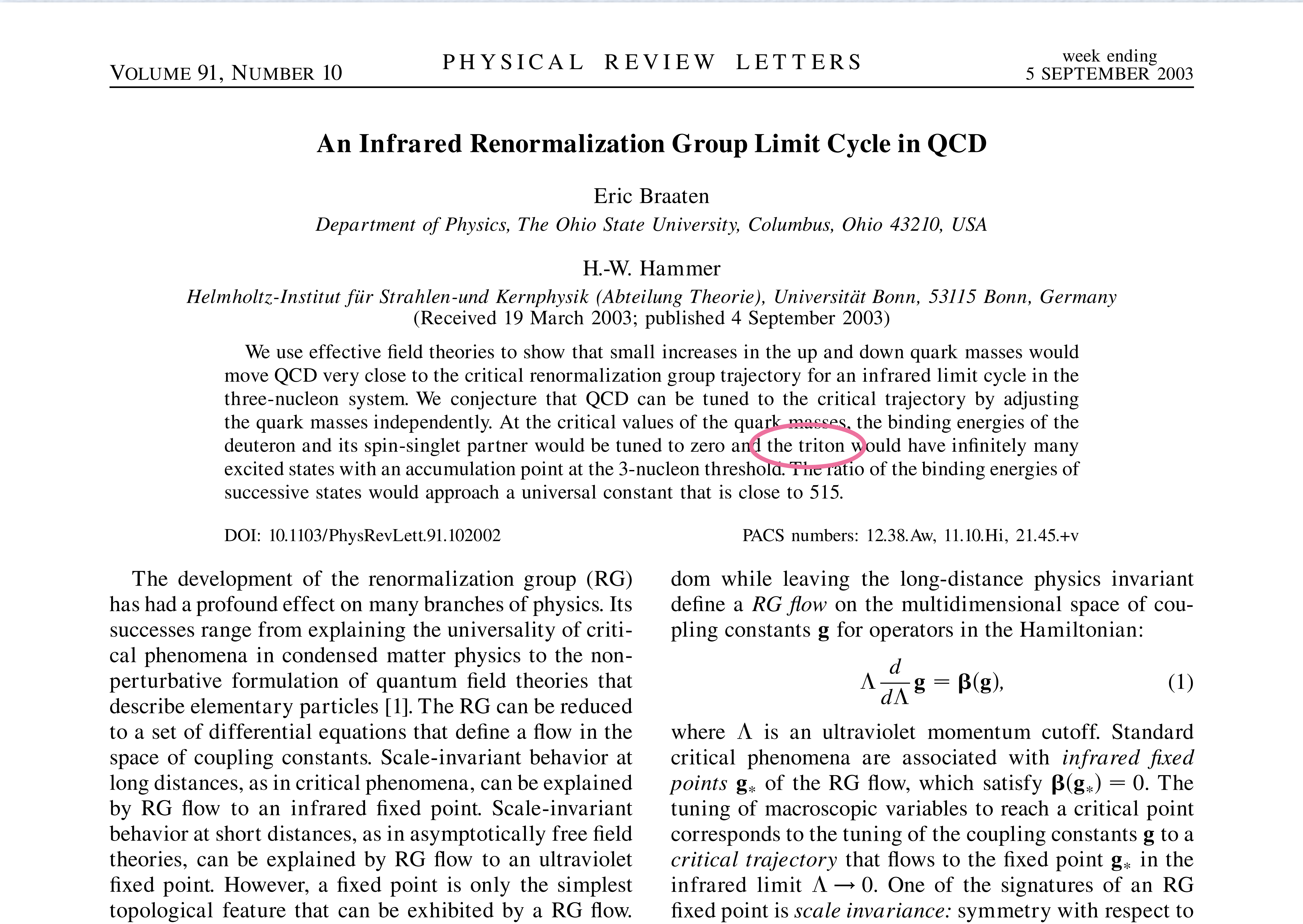
Rice University for ^{7}Li (2009)

Discrete scaling & Universality !

Beyond cold atoms

1. Universality in physics
2. What is the Efimov effect?
3. **Beyond cold atoms: Quantum magnets**
4. Recent progress: Super Efimov effect

Nucleons



VOLUME 91, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2003

An Infrared Renormalization Group Limit Cycle in QCD

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(Received 19 March 2003; published 4 September 2003)

We use effective field theories to show that small increases in the up and down quark masses would move QCD very close to the critical renormalization group trajectory for an infrared limit cycle in the three-nucleon system. We conjecture that QCD can be tuned to the critical trajectory by adjusting the quark masses independently. At the critical values of the quark masses, the binding energies of the deuteron and its spin-singlet partner would be tuned to zero and the triton would have infinitely many excited states with an accumulation point at the 3-nucleon threshold. The ratio of the binding energies of successive states would approach a universal constant that is close to 515.

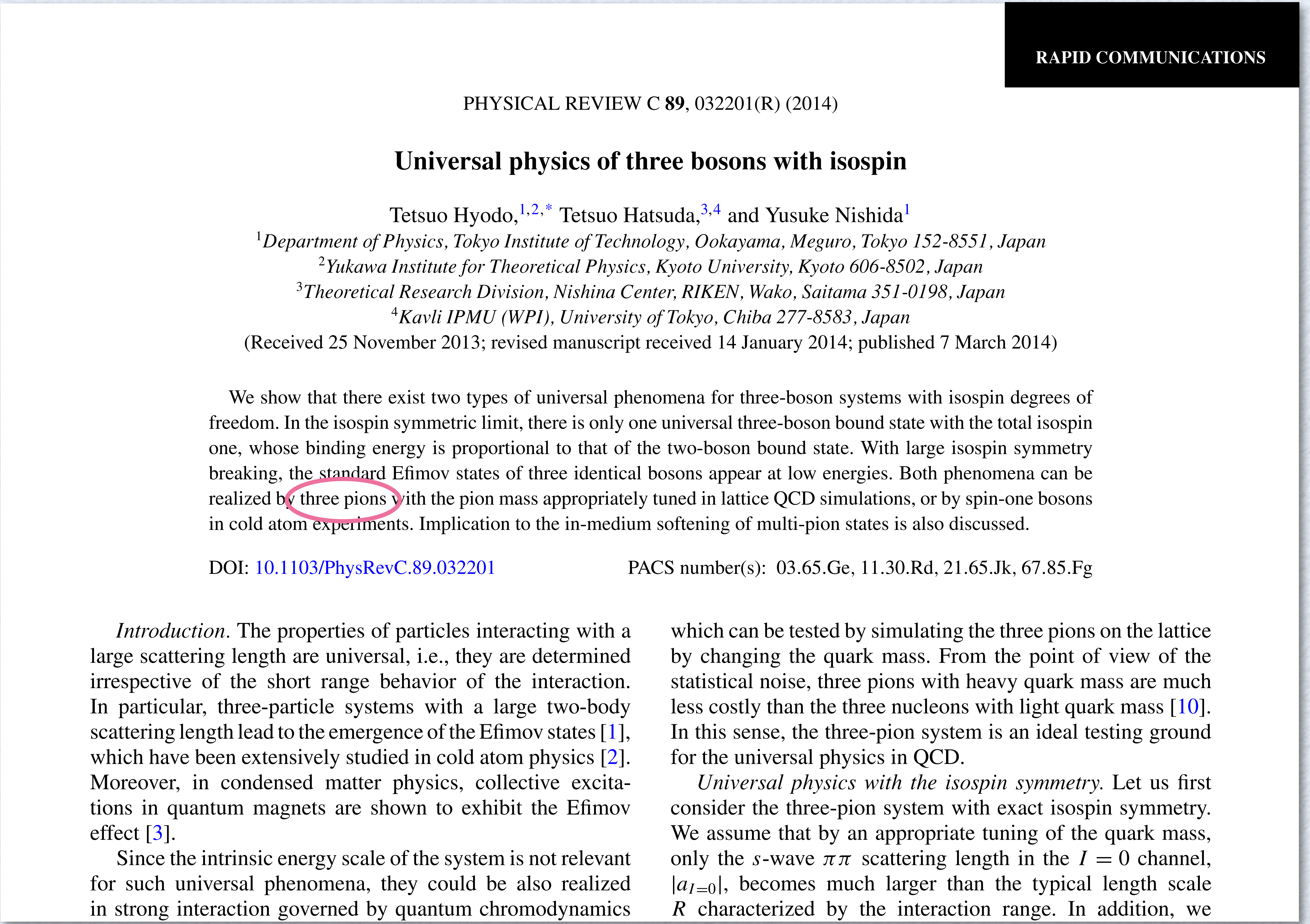
DOI: 10.1103/PhysRevLett.91.102002 PACS numbers: 12.38.Aw, 11.10.Hi, 21.45.+v

The development of the renormalization group (RG) has had a profound effect on many branches of physics. Its successes range from explaining the universality of critical phenomena in condensed matter physics to the nonperturbative formulation of quantum field theories that describe elementary particles [1]. The RG can be reduced to a set of differential equations that define a flow in the space of coupling constants. Scale-invariant behavior at long distances, as in critical phenomena, can be explained by RG flow to an infrared fixed point. Scale-invariant behavior at short distances, as in asymptotically free field theories, can be explained by RG flow to an ultraviolet fixed point. However, a fixed point is only the simplest topological feature that can be exhibited by a RG flow. dom while leaving the long-distance physics invariant define a *RG flow* on the multidimensional space of coupling constants $\mathbf{g}$ for operators in the Hamiltonian:

$$\Lambda \frac{d}{d\Lambda} \mathbf{g} = \boldsymbol{\beta}(\mathbf{g}), \qquad (1)$$

where Λ is an ultraviolet momentum cutoff. Standard critical phenomena are associated with *infrared fixed points* $\mathbf{g}_*$ of the RG flow, which satisfy $\boldsymbol{\beta}(\mathbf{g}_*) = 0$. The tuning of macroscopic variables to reach a critical point corresponds to the tuning of the coupling constants $\mathbf{g}$ to a *critical trajectory* that flows to the fixed point $\mathbf{g}_*$ in the infrared limit $\Lambda \to 0$. One of the signatures of an RG fixed point is *scale invariance:* symmetry with respect to

Pions



RAPID COMMUNICATIONS

PHYSICAL REVIEW C **89**, 032201(R) (2014)

Universal physics of three bosons with isospin

Tetsuo Hyodo,[1,2,*] Tetsuo Hatsuda,[3,4] and Yusuke Nishida[1]
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[2]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[3]*Theoretical Research Division, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*
[4]*Kavli IPMU (WPI), University of Tokyo, Chiba 277-8583, Japan*
(Received 25 November 2013; revised manuscript received 14 January 2014; published 7 March 2014)

We show that there exist two types of universal phenomena for three-boson systems with isospin degrees of freedom. In the isospin symmetric limit, there is only one universal three-boson bound state with the total isospin one, whose binding energy is proportional to that of the two-boson bound state. With large isospin symmetry breaking, the standard Efimov states of three identical bosons appear at low energies. Both phenomena can be realized by three pions with the pion mass appropriately tuned in lattice QCD simulations, or by spin-one bosons in cold atom experiments. Implication to the in-medium softening of multi-pion states is also discussed.

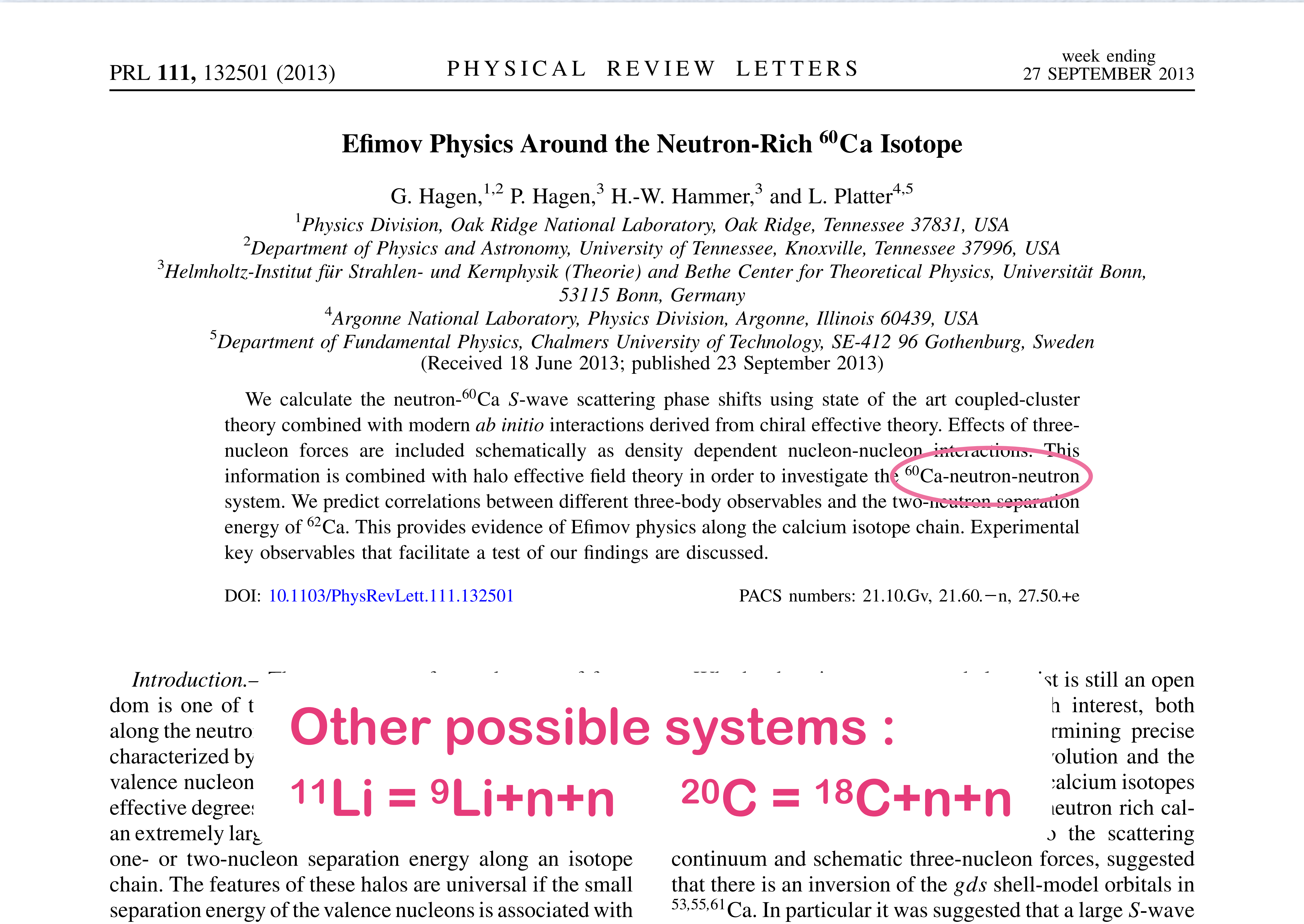
DOI: 10.1103/PhysRevC.89.032201 PACS number(s): 03.65.Ge, 11.30.Rd, 21.65.Jk, 67.85.Fg

Introduction. The properties of particles interacting with a large scattering length are universal, i.e., they are determined irrespective of the short range behavior of the interaction. In particular, three-particle systems with a large two-body scattering length lead to the emergence of the Efimov states [1], which have been extensively studied in cold atom physics [2]. Moreover, in condensed matter physics, collective excitations in quantum magnets are shown to exhibit the Efimov effect [3].

Since the intrinsic energy scale of the system is not relevant for such universal phenomena, they could be also realized in strong interaction governed by quantum chromodynamics which can be tested by simulating the three pions on the lattice by changing the quark mass. From the point of view of the statistical noise, three pions with heavy quark mass are much less costly than the three nucleons with light quark mass [10]. In this sense, the three-pion system is an ideal testing ground for the universal physics in QCD.

Universal physics with the isospin symmetry. Let us first consider the three-pion system with exact isospin symmetry. We assume that by an appropriate tuning of the quark mass, only the s-wave $\pi\pi$ scattering length in the $I = 0$ channel, $|a_{I=0}|$, becomes much larger than the typical length scale R characterized by the interaction range. In addition, we

Halo nuclei



PRL **111,** 132501 (2013) PHYSICAL REVIEW LETTERS week ending 27 SEPTEMBER 2013

Efimov Physics Around the Neutron-Rich ^{60}Ca Isotope

G. Hagen,[1,2] P. Hagen,[3] H.-W. Hammer,[3] and L. Platter[4,5]

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(Received 18 June 2013; published 23 September 2013)

We calculate the neutron-^{60}Ca S-wave scattering phase shifts using state of the art coupled-cluster theory combined with modern *ab initio* interactions derived from chiral effective theory. Effects of three-nucleon forces are included schematically as density dependent nucleon-nucleon interactions. This information is combined with halo effective field theory in order to investigate the ^{60}Ca-neutron-neutron system. We predict correlations between different three-body observables and the two-neutron separation energy of ^{62}Ca. This provides evidence of Efimov physics along the calcium isotope chain. Experimental key observables that facilitate a test of our findings are discussed.

DOI: 10.1103/PhysRevLett.111.132501 PACS numbers: 21.10.Gv, 21.60.−n, 27.50.+e

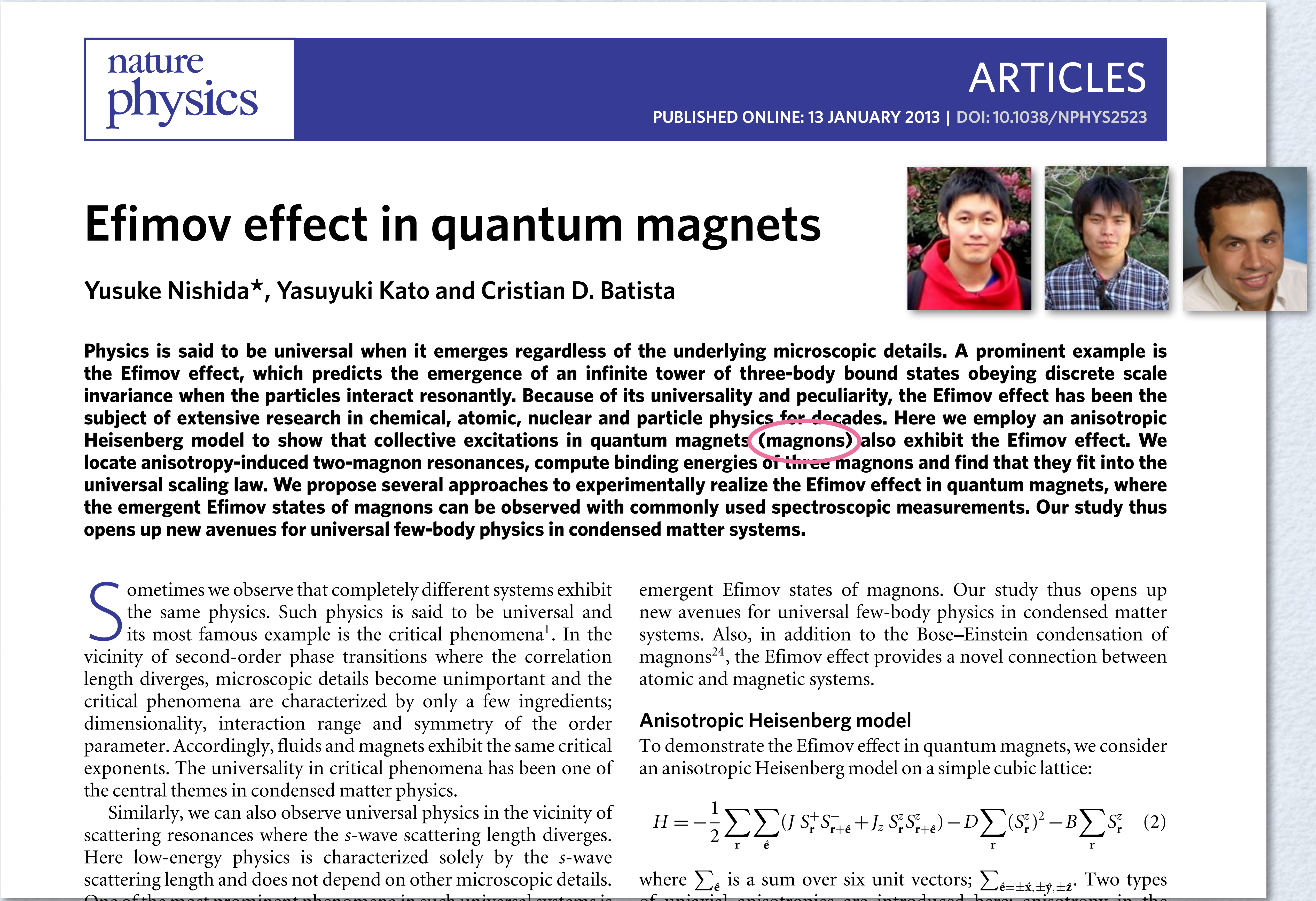
Introduction.–
dom is one of t
along the neutro
characterized by
valence nucleon
effective degree
an extremely lar
one- or two-nucleon separation energy along an isotope chain. The features of these halos are universal if the small separation energy of the valence nucleons is associated with

st is still an open
interest, both
rmining precise
olution and the
alcium isotopes
eutron rich cal-
the scattering
continuum and schematic three-nucleon forces, suggested that there is an inversion of the *gds* shell-model orbitals in ^{53,55,61}Ca. In particular it was suggested that a large S-wave

Other possible systems :

^{11}Li = ^{9}Li+n+n ^{20}C = ^{18}C+n+n

Magnons



nature physics

ARTICLES

PUBLISHED ONLINE: 13 JANUARY 2013 | DOI: 10.1038/NPHYS2523

Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista

Physics is said to be universal when it emerges regardless of the underlying microscopic details. A prominent example is the Efimov effect, which predicts the emergence of an infinite tower of three-body bound states obeying discrete scale invariance when the particles interact resonantly. Because of its universality and peculiarity, the Efimov effect has been the subject of extensive research in chemical, atomic, nuclear and particle physics for decades. Here we employ an anisotropic Heisenberg model to show that collective excitations in quantum magnets (magnons) also exhibit the Efimov effect. We locate anisotropy-induced two-magnon resonances, compute binding energies of three magnons and find that they fit into the universal scaling law. We propose several approaches to experimentally realize the Efimov effect in quantum magnets, where the emergent Efimov states of magnons can be observed with commonly used spectroscopic measurements. Our study thus opens up new avenues for universal few-body physics in condensed matter systems.

Sometimes we observe that completely different systems exhibit the same physics. Such physics is said to be universal and its most famous example is the critical phenomena[1]. In the vicinity of second-order phase transitions where the correlation length diverges, microscopic details become unimportant and the critical phenomena are characterized by only a few ingredients; dimensionality, interaction range and symmetry of the order parameter. Accordingly, fluids and magnets exhibit the same critical exponents. The universality in critical phenomena has been one of the central themes in condensed matter physics.

Similarly, we can also observe universal physics in the vicinity of scattering resonances where the *s*-wave scattering length diverges. Here low-energy physics is characterized solely by the *s*-wave scattering length and does not depend on other microscopic details. One of the most prominent phenomena in such universal systems is

emergent Efimov states of magnons. Our study thus opens up new avenues for universal few-body physics in condensed matter systems. Also, in addition to the Bose–Einstein condensation of magnons[24], the Efimov effect provides a novel connection between atomic and magnetic systems.

Anisotropic Heisenberg model

To demonstrate the Efimov effect in quantum magnets, we consider an anisotropic Heisenberg model on a simple cubic lattice:

$$H=-\frac{1}{2}\sum_{\mathbf{r}}\sum_{\hat{\mathbf{e}}}(J\,S_{\mathbf{r}}^{+}S_{\mathbf{r}+\hat{\mathbf{e}}}^{-}+J_z\,S_{\mathbf{r}}^{z}S_{\mathbf{r}+\hat{\mathbf{e}}}^{z})-D\sum_{\mathbf{r}}(S_{\mathbf{r}}^{z})^2-B\sum_{\mathbf{r}}S_{\mathbf{r}}^{z} \quad (2)$$

where $\sum_{\hat{\mathbf{e}}}$ is a sum over six unit vectors; $\sum_{\hat{\mathbf{e}}=\pm\hat{\mathbf{x}},\pm\hat{\mathbf{y}},\pm\hat{\mathbf{z}}}$. Two types of uniaxial anisotropies are introduced here; anisotropy in the

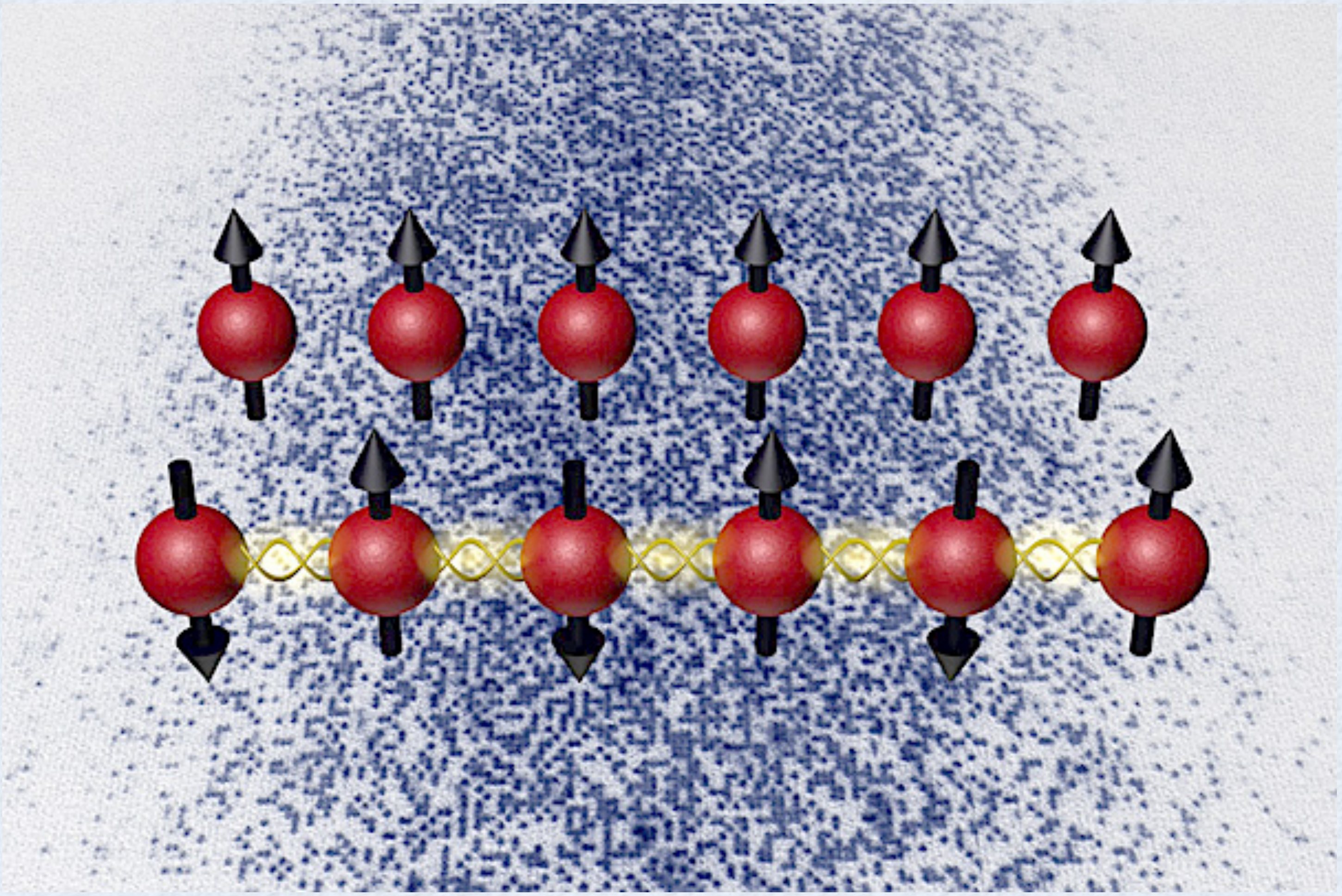
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}\left(J S_r^{+} S_{r+\hat{e}}^{-} + J_z S_r^z S_{r+\hat{e}}^z\right) + D(S_r^z)^2 - B S_r^z\right]$$

exchange anisotropy

single-ion anisotropy



Quantum magnet

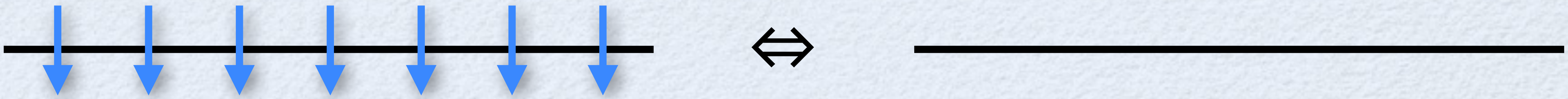
Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(JS_r^+S_{r+\hat{e}}^- + J_zS_r^zS_{r+\hat{e}}^z) + D(S_r^z)^2 - BS_r^z\Big]$$

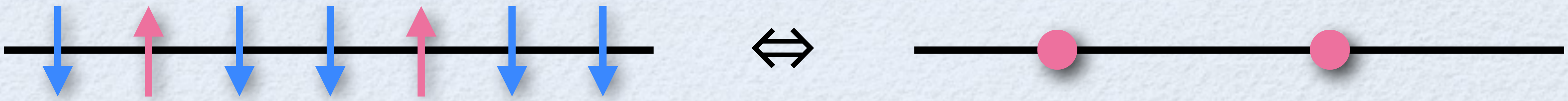
exchange anisotropy

single-ion anisotropy

Spin-boson correspondence



fully polarized state (B→∞) ⇔ No boson = vacuum



N spin-flips ⇔ N bosons = magnons

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}\left(J S_r^{+} S_{r+\hat{e}}^{-} + J_z S_r^z S_{r+\hat{e}}^z\right) + D(S_r^z)^2 - B S_r^z\right]$$

xy-exchange coupling

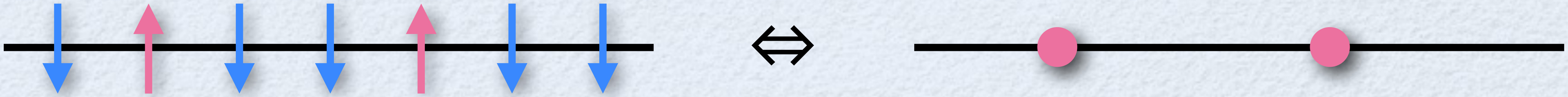
⇔ hopping

single-ion anisotropy

⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction



N spin-flips

N bosons = magnons

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(J S_r^{+} S_{r+\hat{e}}^{-} + J_z S_r^{z} S_{r+\hat{e}}^{z}) + D(S_r^{z})^2 - B S_r^{z}\Big]$$

xy-exchange coupling

⇔ hopping

single-ion anisotropy

⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction

Tune these couplings to induce

scattering resonance between two magnons

⇒ Three magnons show the Efimov effect

Two-magnon resonance

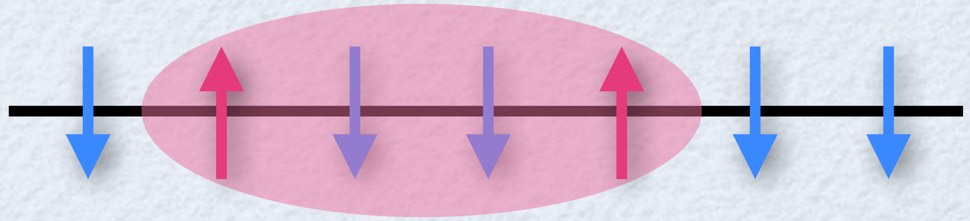
Schrödinger equation for two magnons

$$E\Psi(r_1, r_2) = \Bigg[SJ \sum_{\hat{e}} (2 - \nabla_{1\hat{e}} - \nabla_{2\hat{e}}) \quad \longleftarrow \text{hopping}$$

$$+ J \sum_{\hat{e}} \delta_{r_1,r_2} \nabla_{2\hat{e}} \underbrace{- J_z \sum_{\hat{e}} \delta_{r_1,r_2+\hat{e}} - 2D\delta_{r_1,r_2}}_{\text{neighbor/on-site attraction}} \Bigg] \Psi(r_1, r_2)$$

Scattering length between two magnons

$$\lim_{|r_1 - r_2| \to \infty} \Psi(r_1, r_2)\Big|_{E=0} \to \frac{1}{|r_1 - r_2|} - \frac{1}{a_s}$$



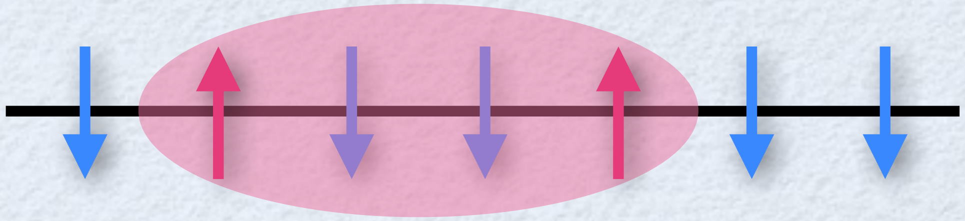
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

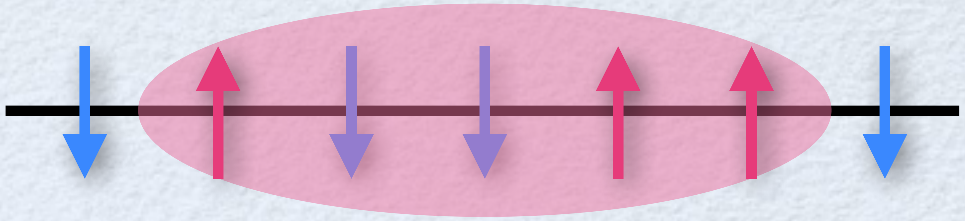
➡ Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- Spin-1/2

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- Spin-1, D=0

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- Spin-1, $J_z=J>0$

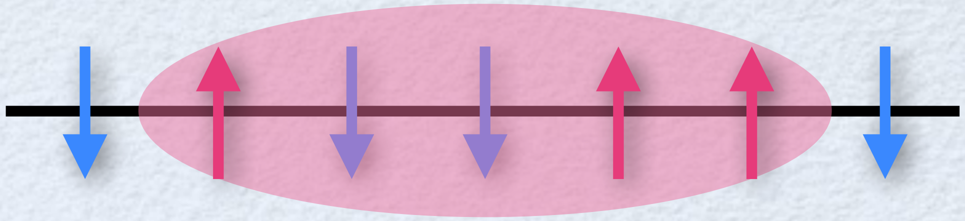
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- Spin-1, $J_z=J<0$

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

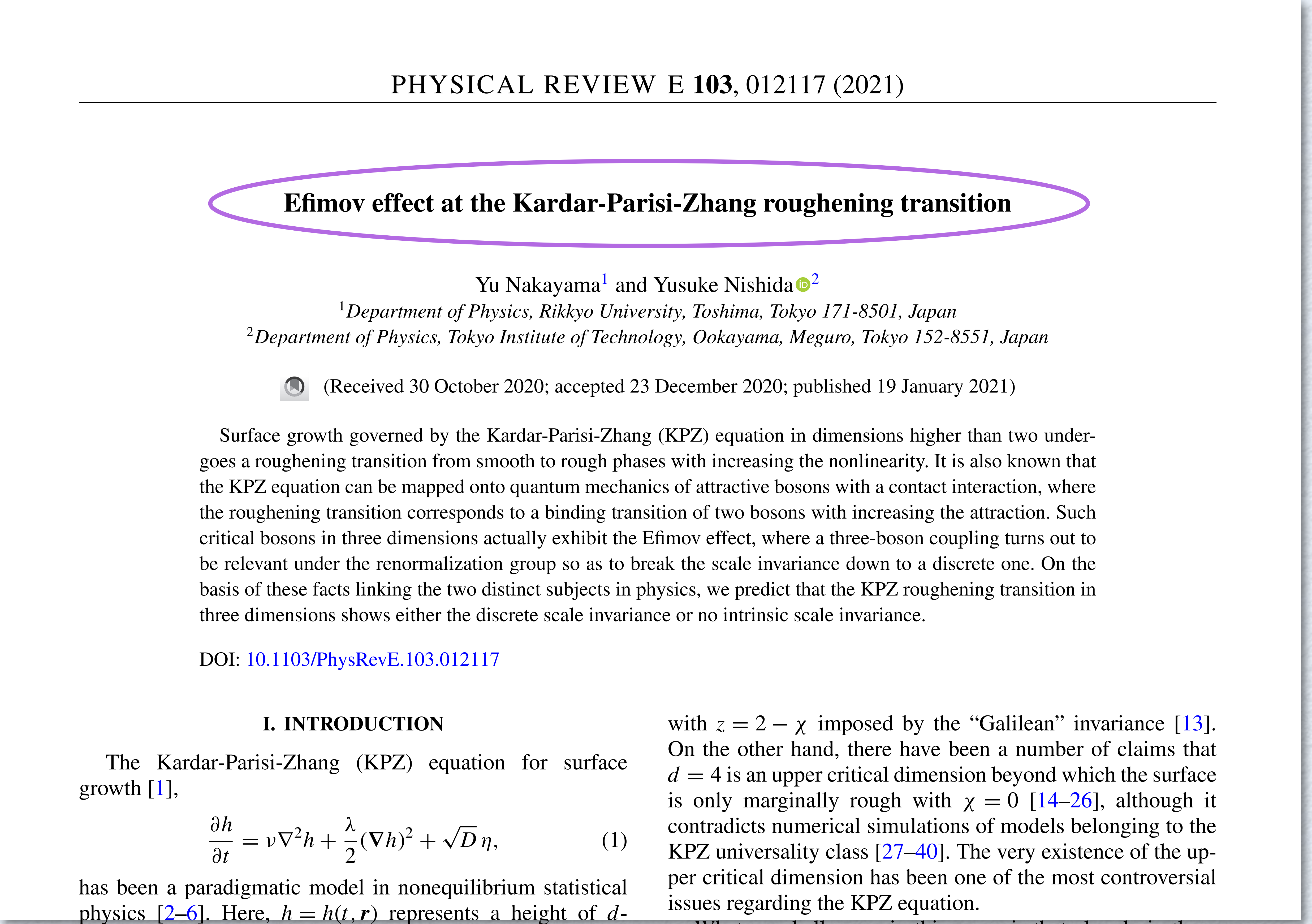
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

➡ **Universal scaling law by ~ 22.7 confirms they are Efimov states !**

KPZ roughening transition



PHYSICAL REVIEW E **103**, 012117 (2021)

Efimov effect at the Kardar-Parisi-Zhang roughening transition

Yu Nakayama[1] and Yusuke Nishida[2]
[1] *Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan*
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(Received 30 October 2020; accepted 23 December 2020; published 19 January 2021)

Surface growth governed by the Kardar-Parisi-Zhang (KPZ) equation in dimensions higher than two undergoes a roughening transition from smooth to rough phases with increasing the nonlinearity. It is also known that the KPZ equation can be mapped onto quantum mechanics of attractive bosons with a contact interaction, where the roughening transition corresponds to a binding transition of two bosons with increasing the attraction. Such critical bosons in three dimensions actually exhibit the Efimov effect, where a three-boson coupling turns out to be relevant under the renormalization group so as to break the scale invariance down to a discrete one. On the basis of these facts linking the two distinct subjects in physics, we predict that the KPZ roughening transition in three dimensions shows either the discrete scale invariance or no intrinsic scale invariance.

DOI: 10.1103/PhysRevE.103.012117

I. INTRODUCTION

The Kardar-Parisi-Zhang (KPZ) equation for surface growth [1],

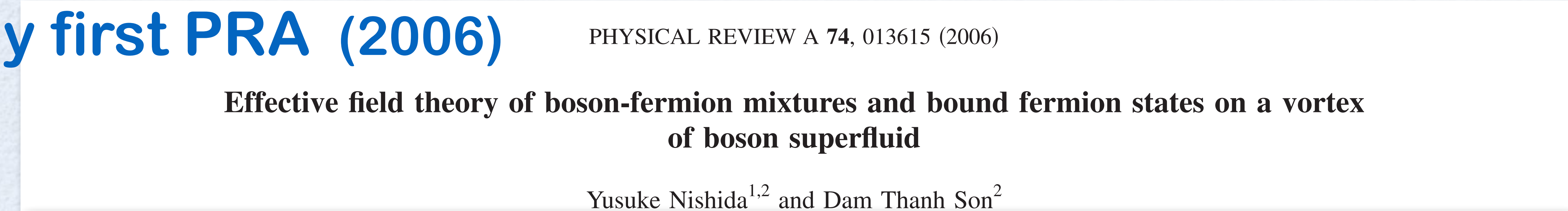
$$\frac{\partial h}{\partial t} = \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \sqrt{D}\,\eta, \tag{1}$$

has been a paradigmatic model in nonequilibrium statistical physics [2–6]. Here, $h = h(t, \boldsymbol{r})$ represents a height of d-

with $z = 2 - \chi$ imposed by the "Galilean" invariance [13]. On the other hand, there have been a number of claims that $d = 4$ is an upper critical dimension beyond which the surface is only marginally rough with $\chi = 0$ [14–26], although it contradicts numerical simulations of models belonging to the KPZ universality class [27–40]. The very existence of the upper critical dimension has been one of the most controversial issues regarding the KPZ equation.

PhysRevABCDE completed !

My first PRA (2006)

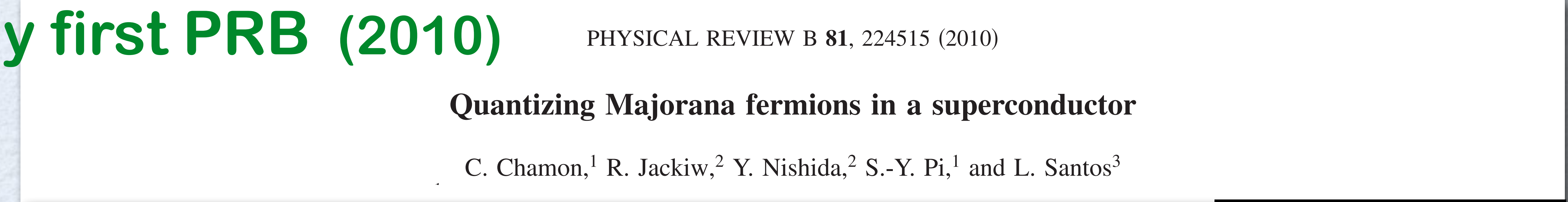


PHYSICAL REVIEW A **74**, 013615 (2006)

Effective field theory of boson-fermion mixtures and bound fermion states on a vortex of boson superfluid

Yusuke Nishida[1,2] and Dam Thanh Son[2]

My first PRB (2010)

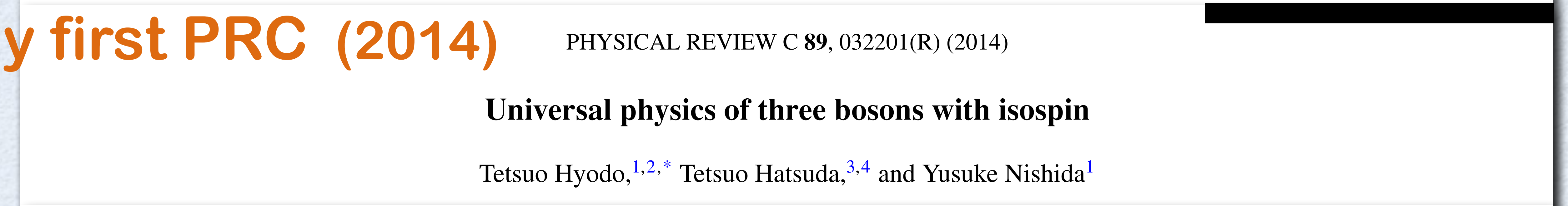


PHYSICAL REVIEW B **81**, 224515 (2010)

Quantizing Majorana fermions in a superconductor

C. Chamon,[1] R. Jackiw,[2] Y. Nishida,[2] S.-Y. Pi,[1] and L. Santos[3]

My first PRC (2014)

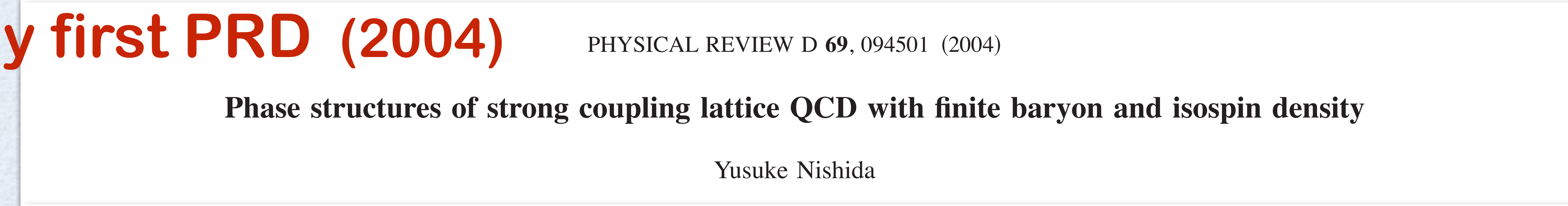


PHYSICAL REVIEW C **89**, 032201(R) (2014)

Universal physics of three bosons with isospin

Tetsuo Hyodo,[1,2,*] Tetsuo Hatsuda,[3,4] and Yusuke Nishida[1]

My first PRD (2004)

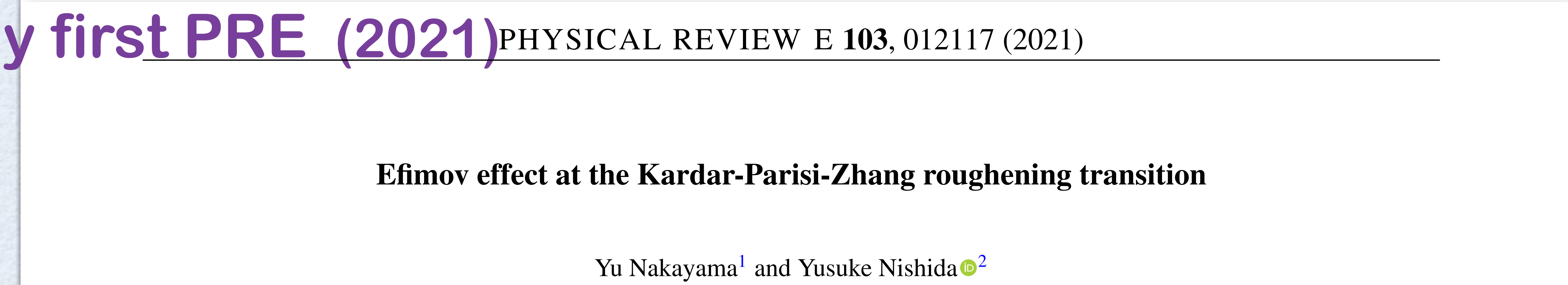


PHYSICAL REVIEW D **69**, 094501 (2004)

Phase structures of strong coupling lattice QCD with finite baryon and isospin density

Yusuke Nishida

My first PRE (2021)



PHYSICAL REVIEW E **103**, 012117 (2021)

Efimov effect at the Kardar-Parisi-Zhang roughening transition

Yu Nakayama[1] and Yusuke Nishida[2]

Recent progress

1. Universality in physics
2. What is the Efimov effect ?
3. Beyond cold atoms: Quantum magnets
4. **Recent progress: Super Efimov effect**

Few-body universality

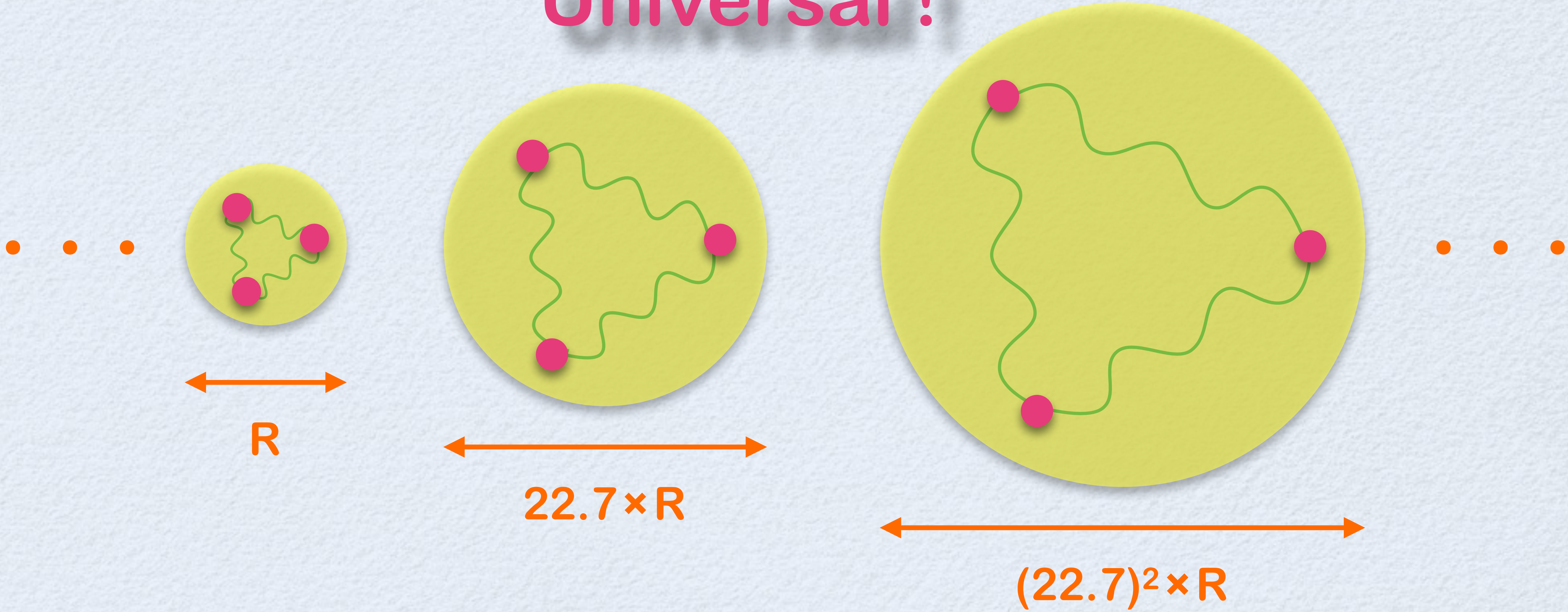
Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

→ Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Universal !



Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

Infinite bound states with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Efimov effect in other systems ?

No, only in 3D with s-wave resonance

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x	x
1D	x	x	

Y.N. & S.Tan, Few-Body Syst

Y.N. & D.Lee Phys Rev A

Few-body universality

Efimov effect (1970)

- 3 bosons
- 3 dimensions
- s-wave resonance

Infinite bound states
with exponential scaling

$$E_n \sim e^{-2\pi n}$$

Different universality in other systems ?

Yes, super Efimov effect in 2D with p-wave !

	s-wave	p-wave	d-wave
3D	O	x	x
2D	x	x !!!	x
1D	x	x	

Y.N. & S.Tan,
Few-Body Syst

Y.N. & D.Lee
Phys Rev A

Few-body universality

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

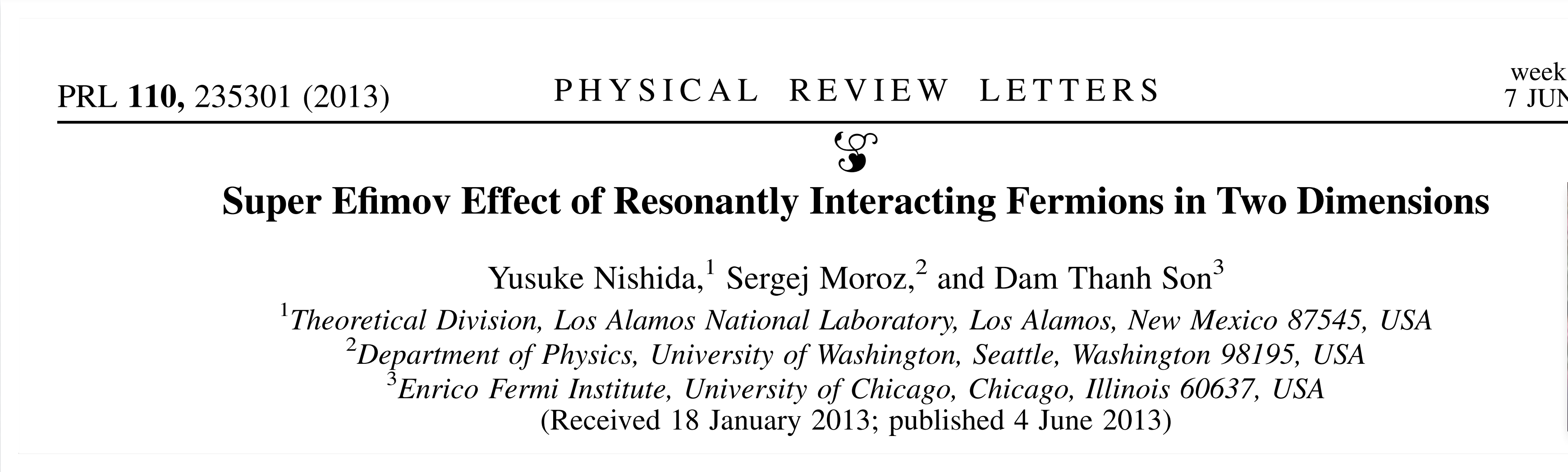
Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

"doubly" exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$



PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

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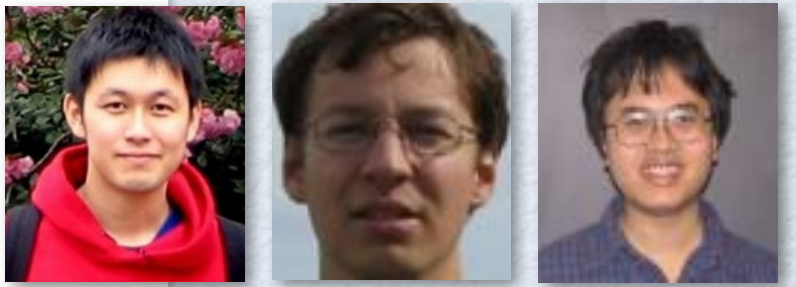
(Received 18 January 2013; published 4 June 2013)



Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

nuclear physics	atomic physics	condensed matter
prediction (1970)	realization (2006)	proposal (2013)

✓ Efimov effect in quantum magnets

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ Novel universality: Super Efimov effect

Y.N, S.M, D.T.S, Phys Rev Lett 110, 235301 (2013)