エフィモフ効果と普遍性

西田 祐介（東工大）

2014年2月5-6日

集中講義@基研

Plan of this talk

1. Universality in physics
2. What is the Efimov effect?

   Keywords: universality, scale invariance quantum anomaly, RG limit cycle

3. Efimov effect in quantum magnets*
4. New progress: Super Efimov effect

nature physics

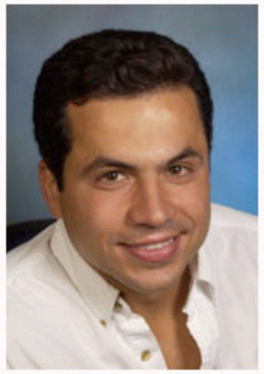
ARTICLES

PUBLISHED ONLINE: 13 JANUARY 2013 | DOI: 10.1038/NPHYS2523

Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista

Physics is said to be universal when it emerges regardless of the underlying microscopic details. A prominent example is



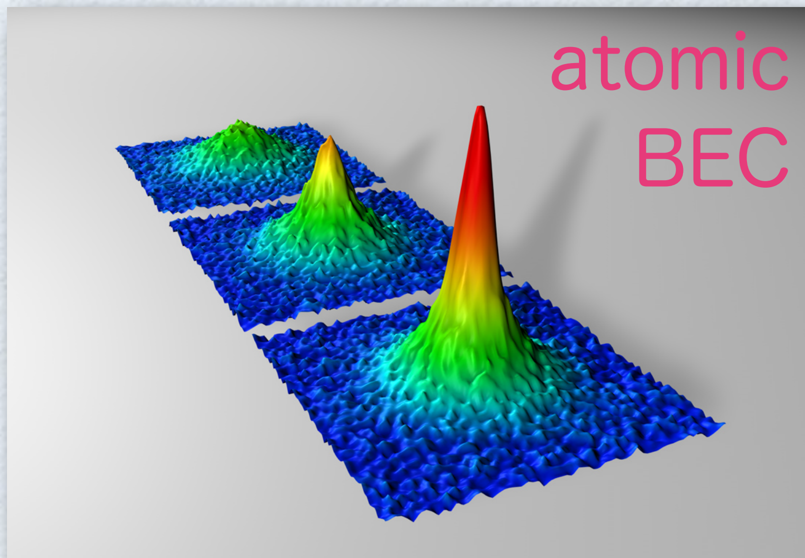
Cristian Batista

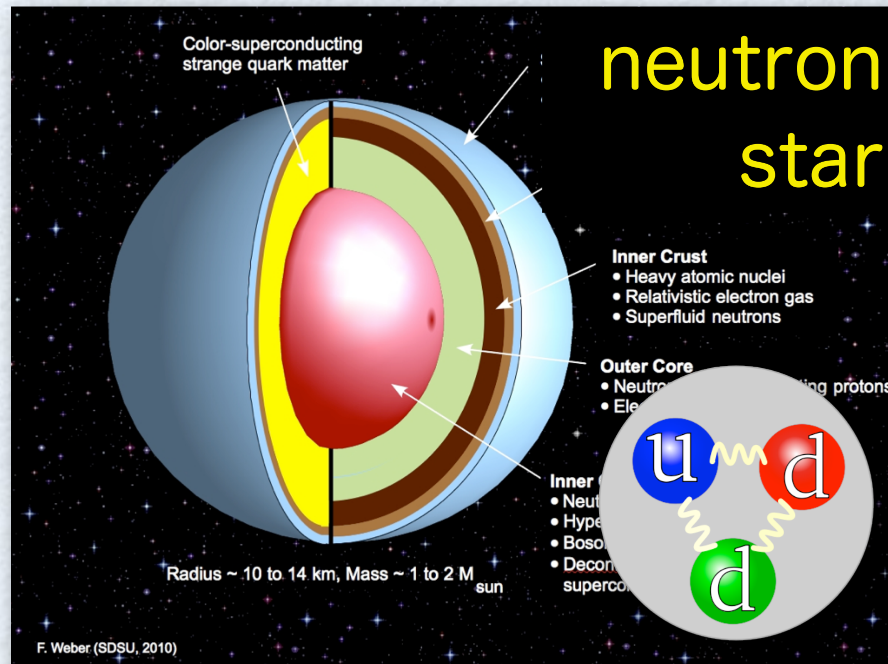
Introduction

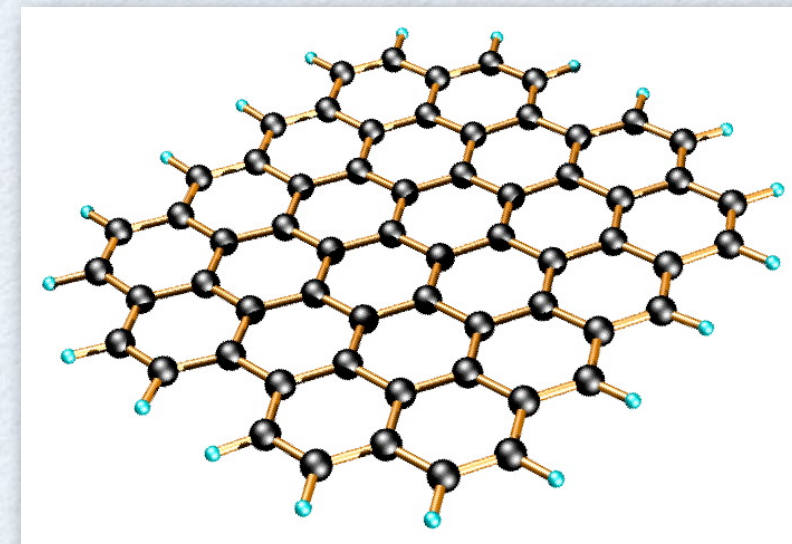
1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. New progress: Super Efimov effect

(ultimate) Goal of research

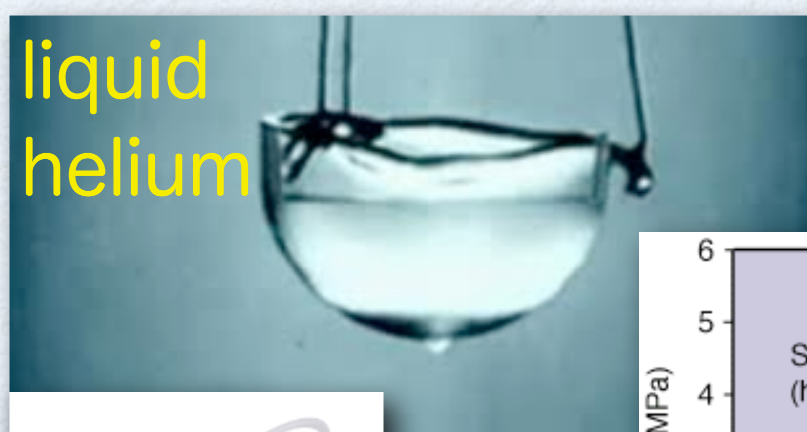
Understand physics of few and many particles governed by quantum mechanics

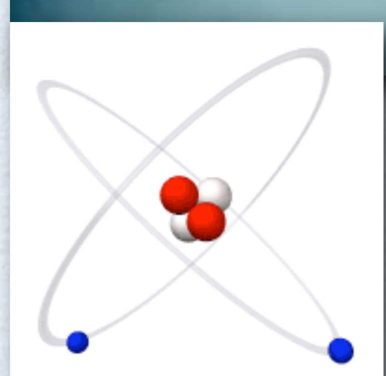


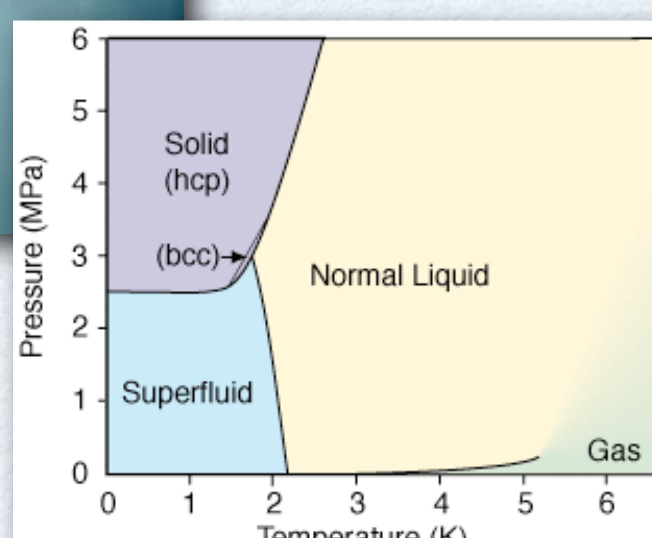




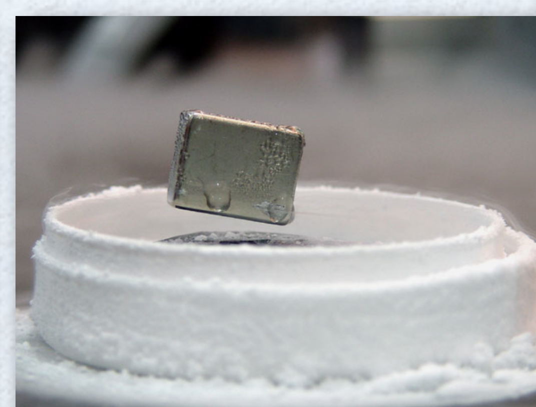
graphene

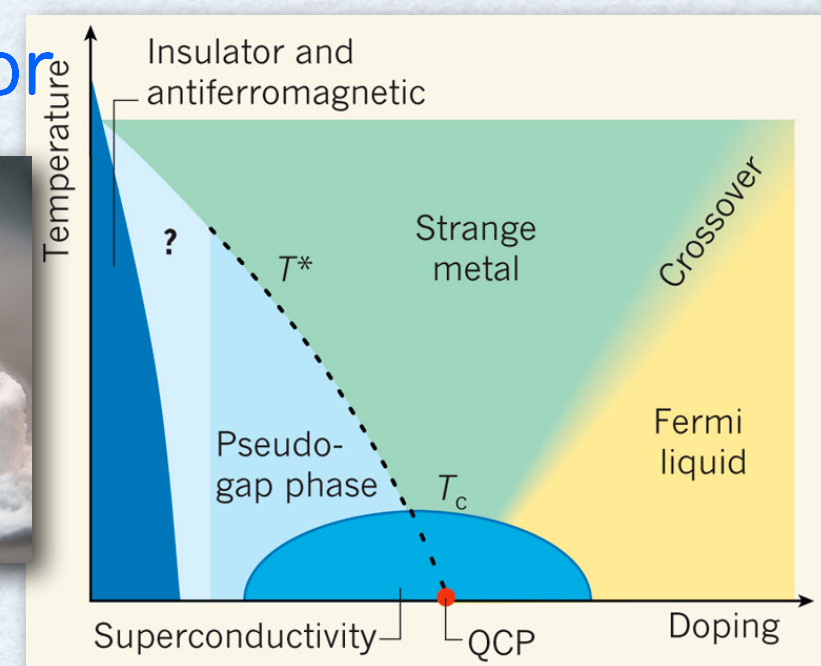






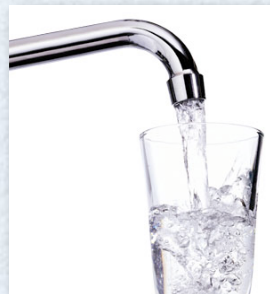
superconductor

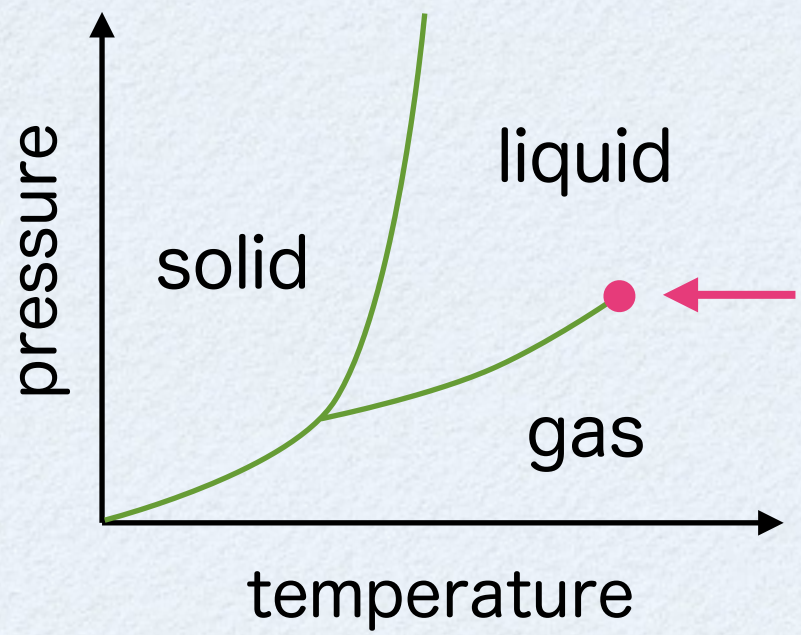


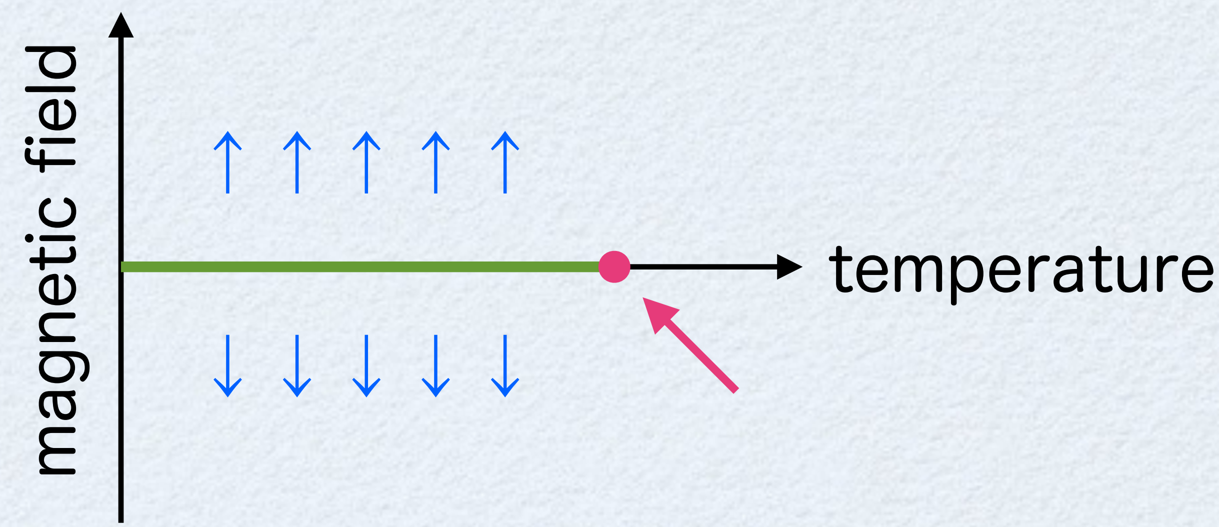


When physics is universal ?

A1. Continuous phase transitions ⇔ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 



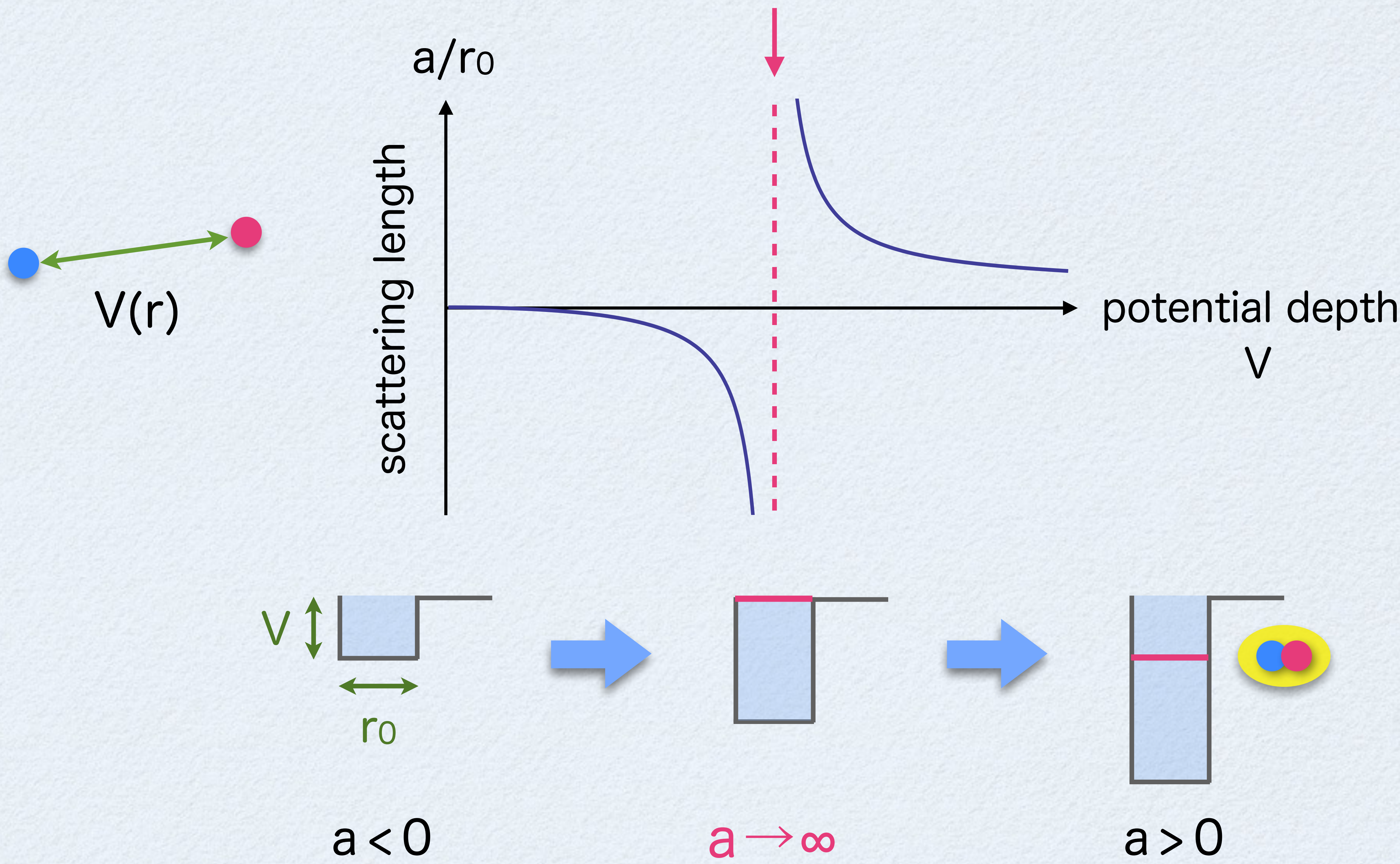


Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\rm liq} - \rho_{\rm gas} \sim (T_{\rm c} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\rm c} - T)^{\beta}$$

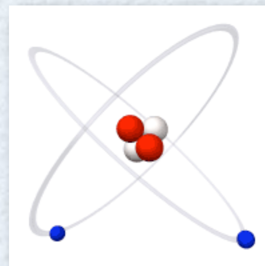
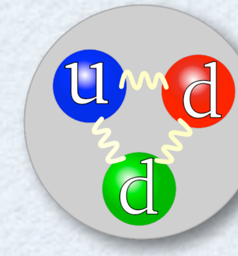
When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$



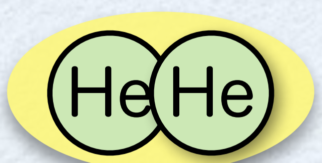
When physics is universal ?

A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$

E.g. ^{4}He atoms  vs. proton/neutron 

van der Waals force:

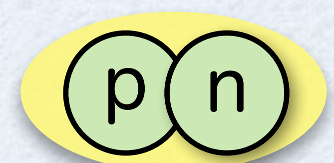
$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$



$E_{\text{binding}} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{\text{binding}} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\text{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

➡ Physics only depends on the scattering length “a”

Efimov effect

1. Universality in physics
2. **What is the Efimov effect?**
3. Efimov effect in quantum magnets
4. New progress: Super Efimov effect

Efimov effect

Efimov (1970)

Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

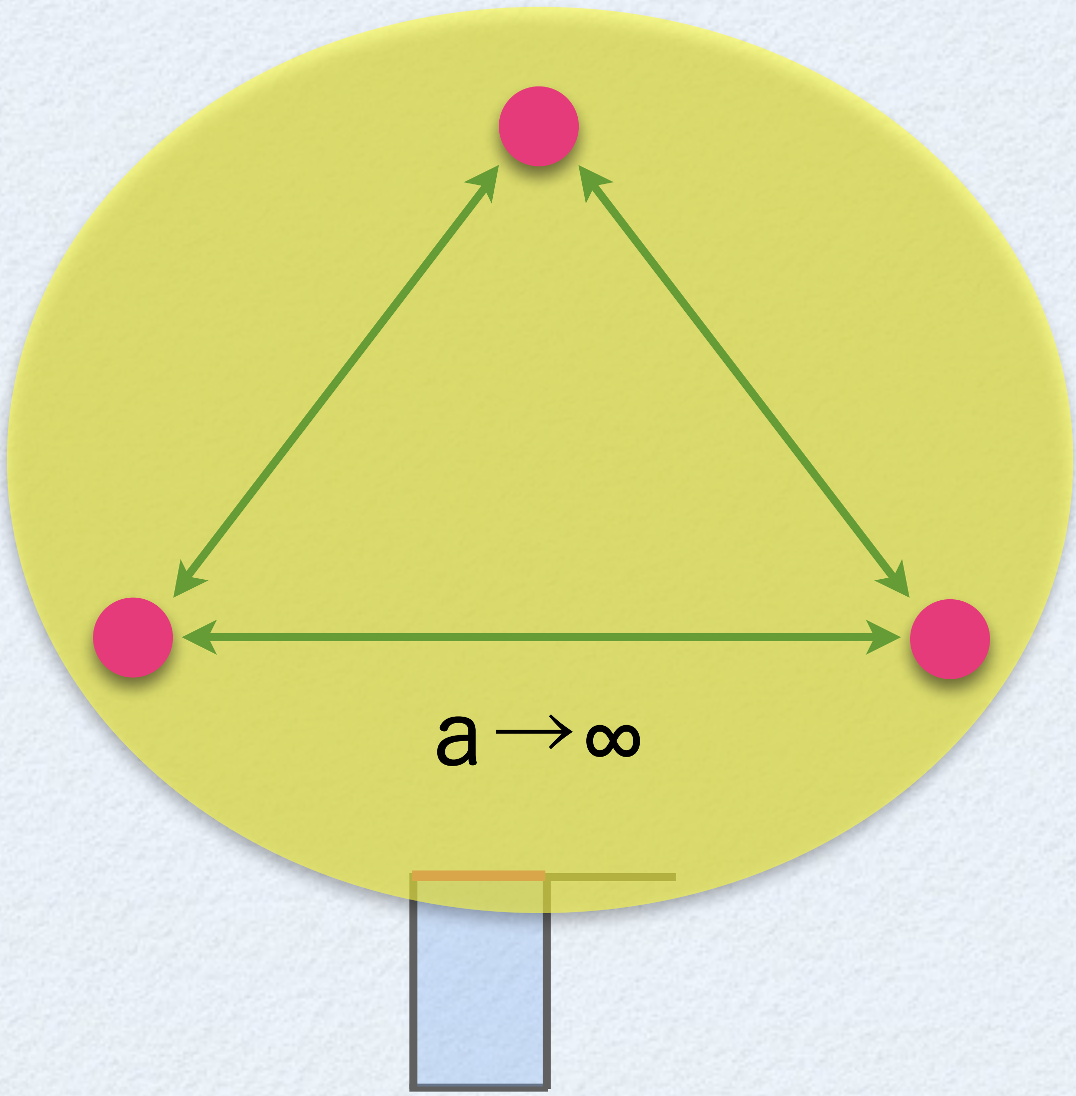
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

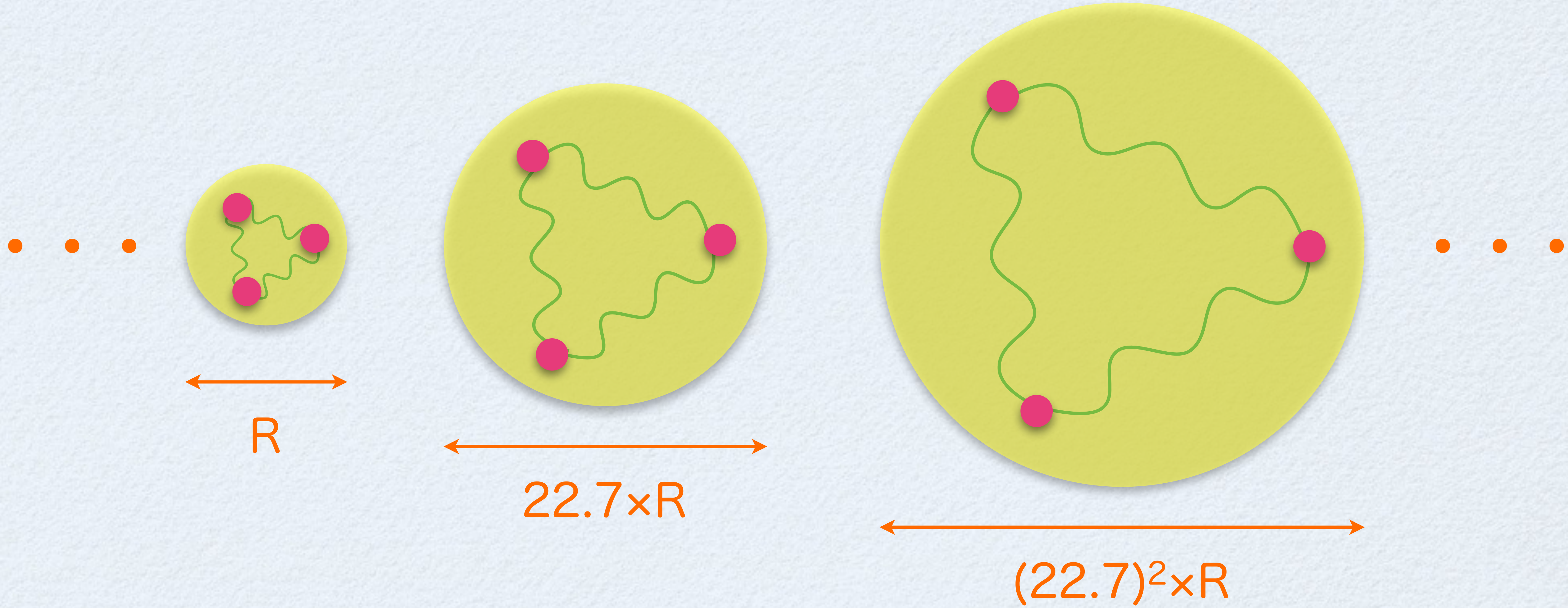
Efimov (1970)



Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



Discrete scaling symmetry

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)

• • •

• • •

Discrete scaling symmetry

Why Efimov effect happens ?

Keywords

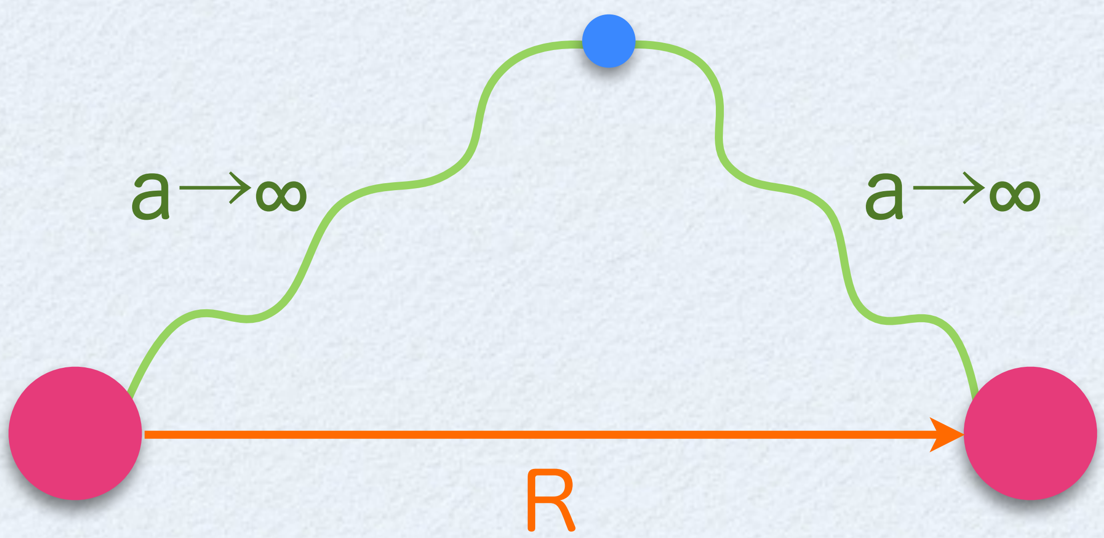
✓ Universality

- Scale invariance
- Quantum anomaly
- RG limit cycle

Why Efimov effect happens ?

Two heavy (M) and one light (m) particles

➡ Born-Oppenheimer approximation



Binding energy of a light particle

$$E_b(R) = -\boxed{\frac{\hbar^2}{2mR^2}} \times (0.5671\ldots)^2$$

Scale invariance at a→∞

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial \boldsymbol{R}^2} + V(R)\right]\psi(\boldsymbol{R}) = -\frac{\hbar^2\kappa^2}{M}\psi(\boldsymbol{R}) \qquad V(R) \equiv E_b(R)$$

Why Efimov effect happens ?

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\left(\frac{\partial^2}{\partial R^2}+\frac{2}{R}\frac{\partial}{\partial R}\right)-\frac{\hbar^2}{2mR^2}(0.5671\ldots)^2\right]\psi(R)=-\frac{\hbar^2\kappa^2}{M}\psi(R)$$

$$\psi(R)=R^{-1/2}K_{i\alpha}(\kappa R) \qquad \alpha^2\equiv\frac{M}{2m}(0.5671\ldots)^2-\frac{1}{4}$$

$$\to R^{-1/2}\sin[\alpha\ln(\kappa R)+\delta] \qquad (R\to 0)$$

ψ'/ψ has to be fixed by short-range physics

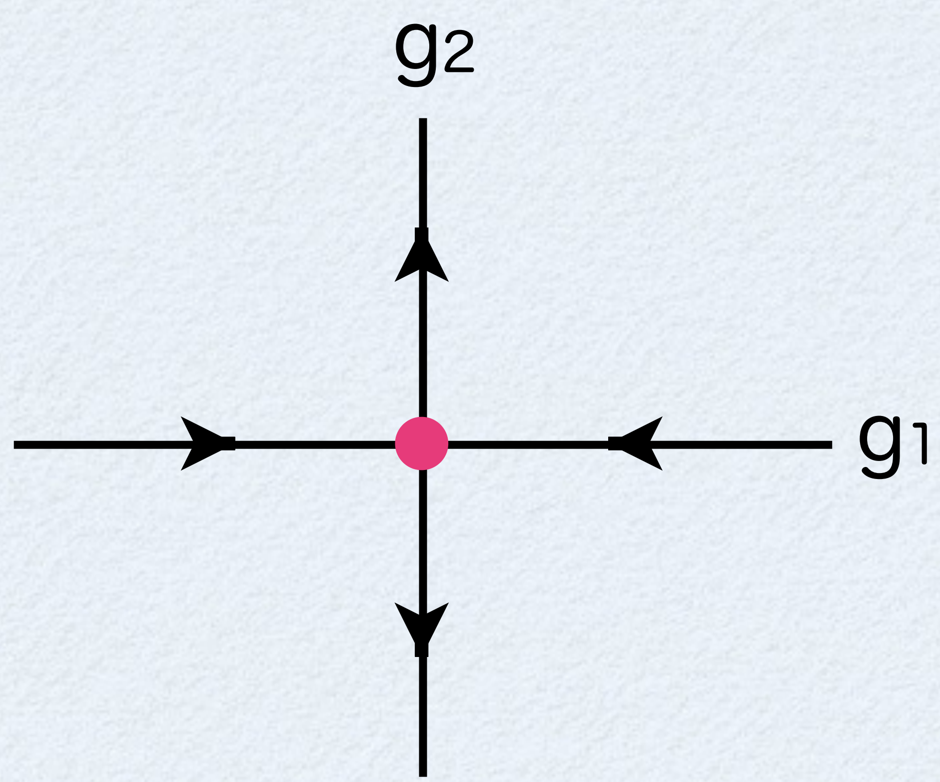
➡ If $\kappa=\kappa_*$ is a solution, $\kappa=(e^{\pi/\alpha})^n\kappa_*$ are solutions!

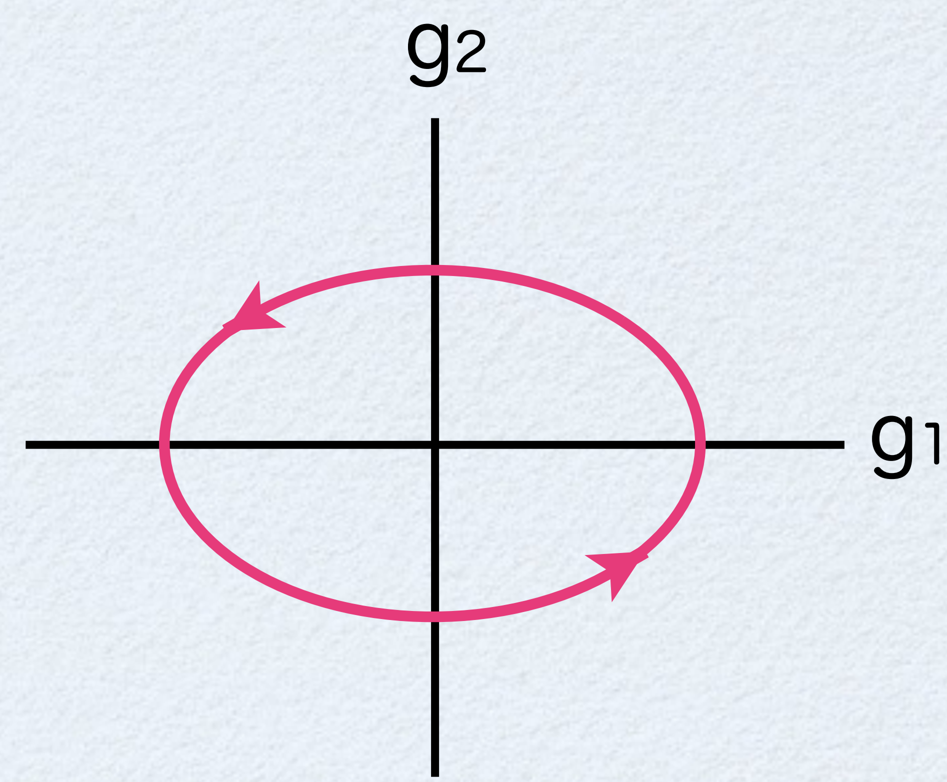
Classical scale invariance is broken by κ_*

= Quantum anomaly

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

RG limit cycle

⇒ Discrete scale invariance

E.g. E????v effect

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

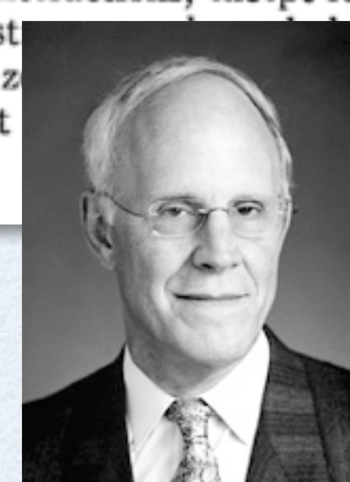
KENNETH G. WILSON
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†

(Received 30 November 1970)

The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of st
must break a symmetry of the combined interactions with z
weak interactions can be understood assuming only that
interactions.

QCD is asymptotic free

(2004 Nobel prize)



Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

KENNETH G. WILSON
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†
(Received 30 November 1970)

The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of strong, weak, and electromagnetic interacti must break a symmetry of the combined interactions with zero mass. (4) The $\Delta I=\frac{1}{2}$ rule in nonlepto weak interactions can be understood assuming only that a renormalization group exists for str interactions.

Efimov effect (1970) is its rare manifestation!

Effective field theory

PHYSICAL REVIEW LETTERS

VOLUME 82 | 18 JANUARY 1999 | NUMBER 3

Renormalization of the Three-Body System with Short-Range Interactions

P. F. Bedaque,[1,*] H.-W. Hammer,[2,†] and U. van Kolck[3,4,‡]

[1]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195*
[2]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*
[3]*Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125*
[4]*Department of Physics, University of Washington, Seattle, Washington 98195*
(Received 9 September 1998)

We discuss renormalization of the nonrelativistic three-body problem with short-range forces. The problem becomes nonperturbative at momenta of the order of the inverse of the two-body scattering length, and an infinite number of graphs must be summed. This summation leads to a cutoff dependence that does not appear in any order in perturbation theory. We argue that this cutoff dependence can be absorbed in a single three-body counterterm and compute the running of the three-body force with the cutoff. We comment on the relevance of this result for the effective field theory program in nuclear and molecular physics. [S0031-9007(98)08276-3]

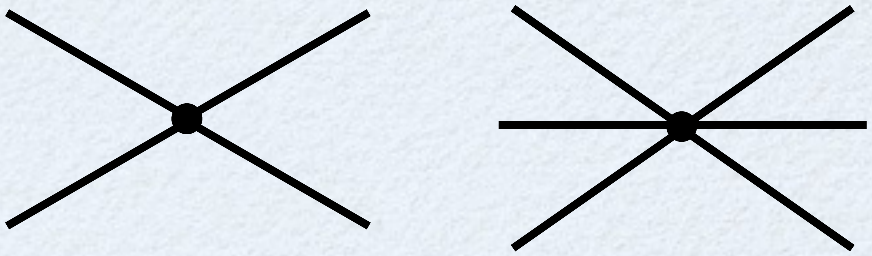
PACS numbers: 03.65.Nk, 11.80.Jy, 21.45.+v, 34.20.Gj

Systems composed of particles with momenta k much

dence can be absorbed in the coefficients of the leading-

Effective field theory

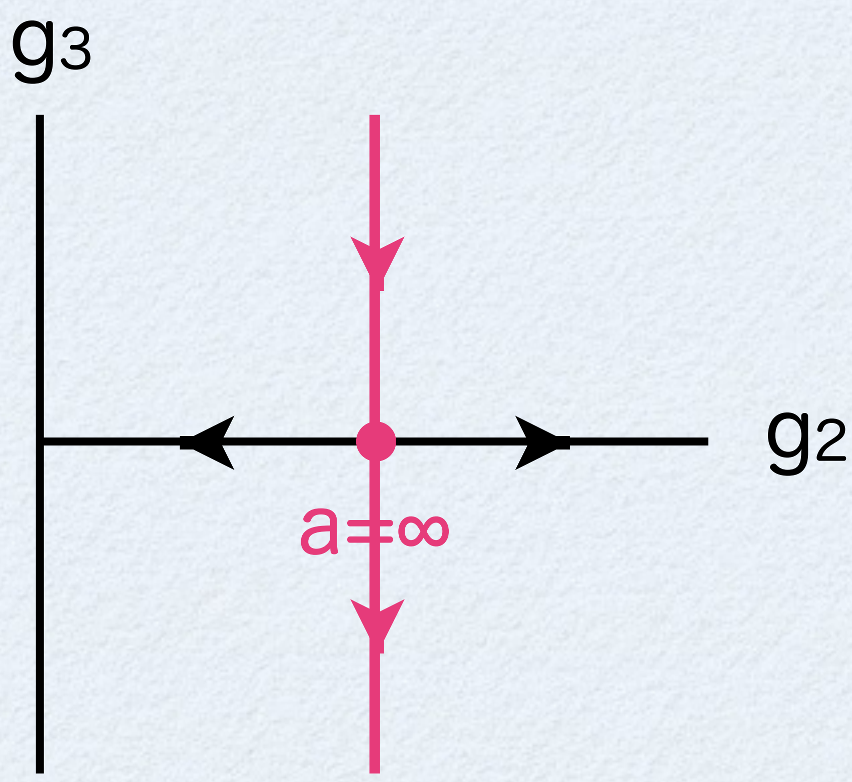
$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + g_2(\psi^\dagger\psi)^2 + g_3(\psi^\dagger\psi)^3$$



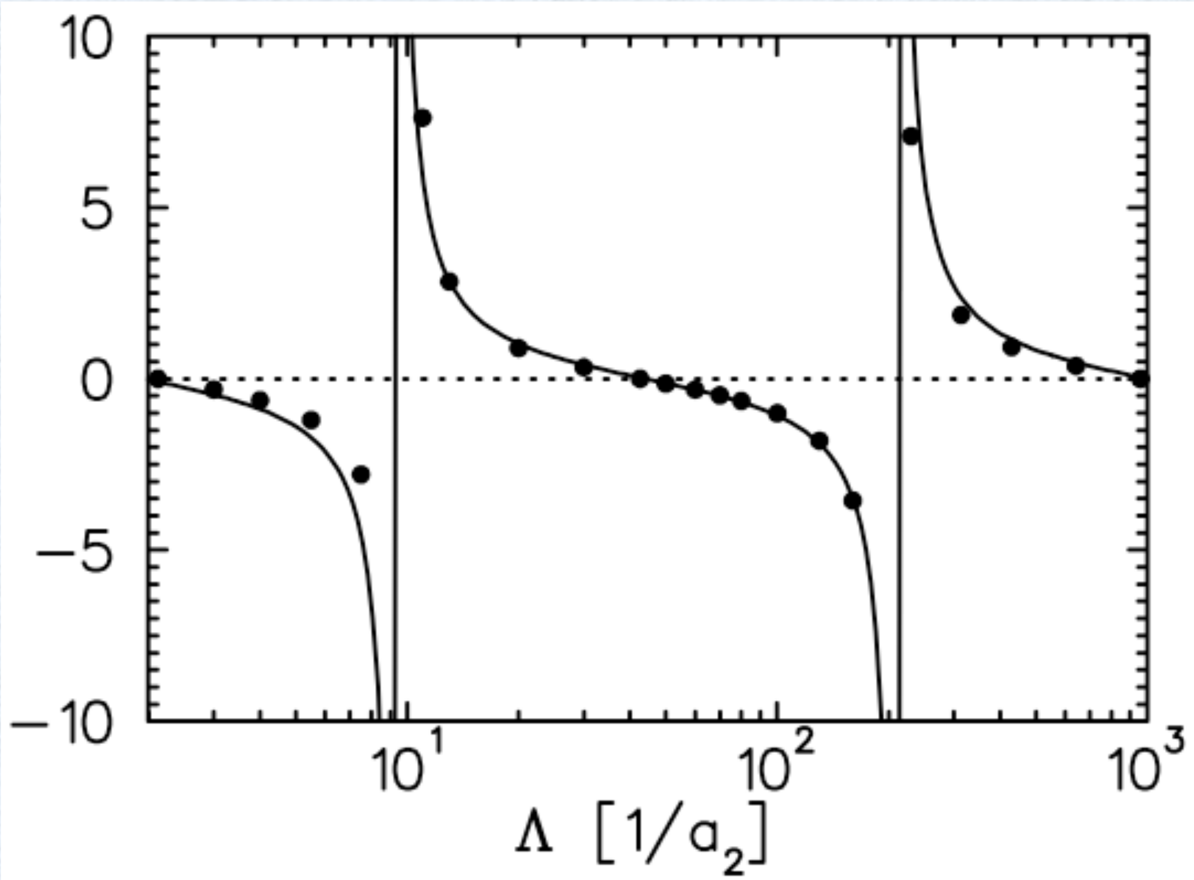
g_2 has a fixed point corresponding to a=∞

What is flow of g_3?

$$g_3(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$



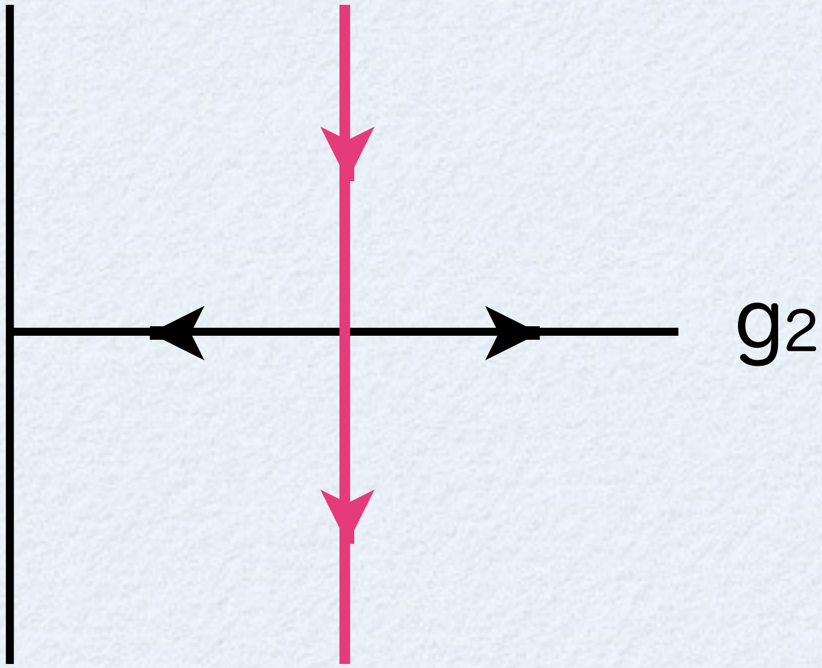
RG limit cycle



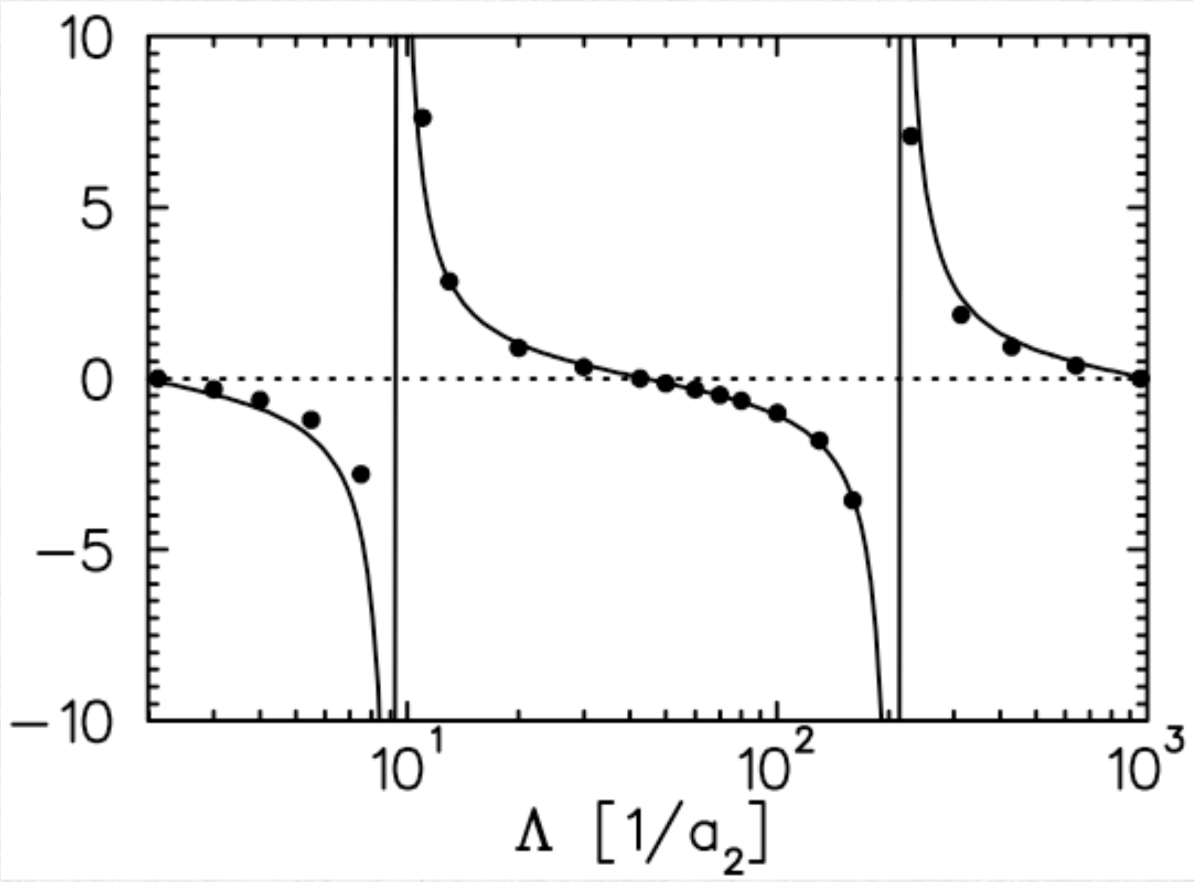
Effective field theory

What is flow of g_3?

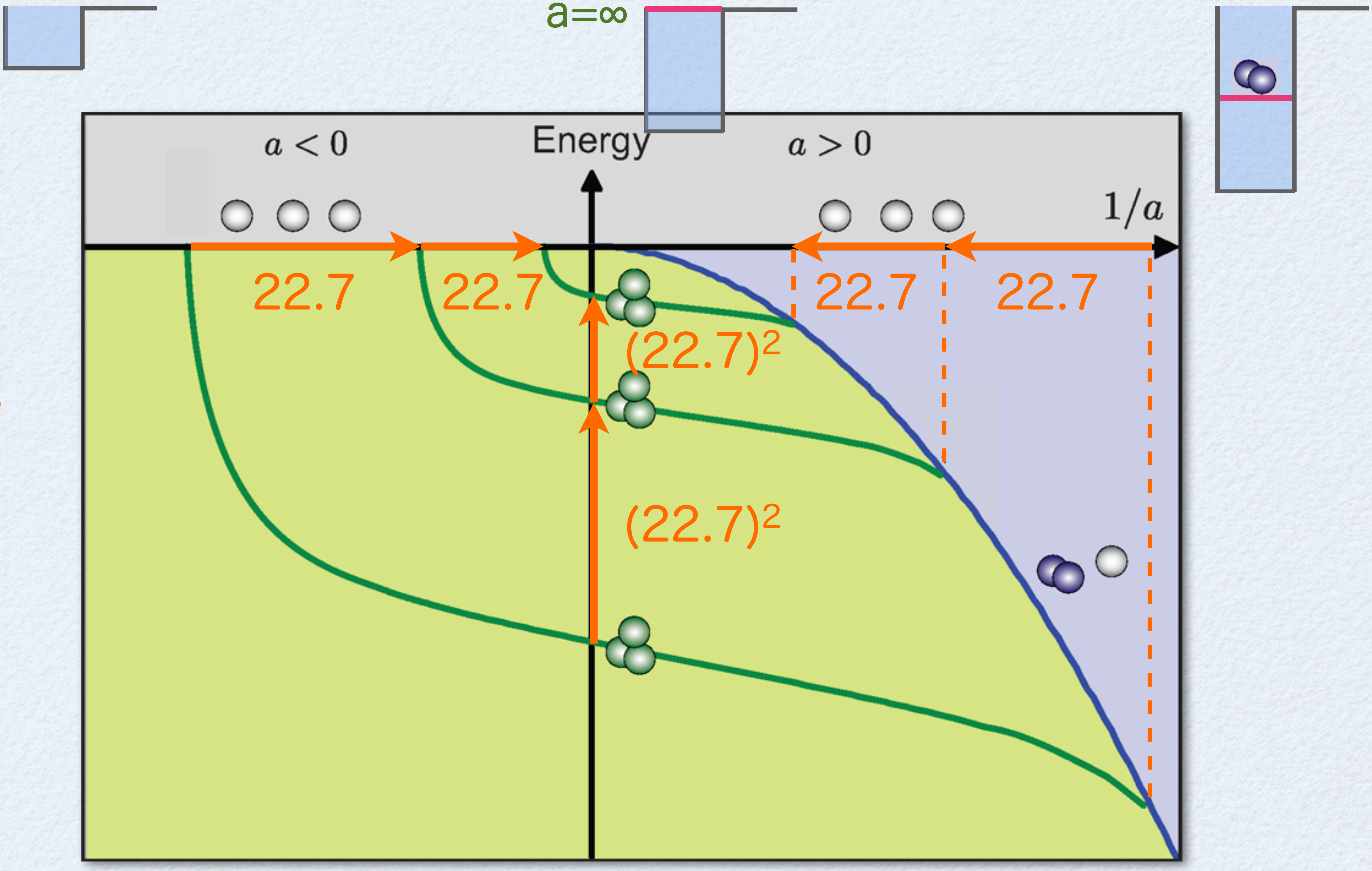
$$g_3(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$



RG limit cycle



Efimov effect at a≠∞



Ferlaino & Grimm, Physics (2010)

Discrete scaling symmetry

Why 22.7 ?

Just a numerical number given by

22.694382595366695192860217 1369...

log(22.694382595366695192860217 1369...)

= 3.12221174311042196807309 1732438...

= π / 1.00623782510278148906406681234...

= π / s_0

$$\frac{2\pi}{s_0}\frac{\sinh(\frac{\pi}{6}s_0)}{\cosh(\frac{\pi}{2}s_0)}=\frac{\sqrt{3}\pi}{4}$$

22.7 = exp(π / 1.006...)

Where Efimov effect appears ?

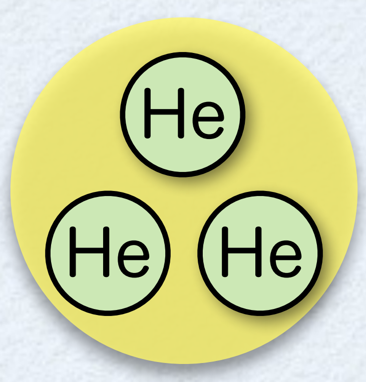
× Originally, Efimov considered

^{3}H nucleus (≈3n) and ^{12}C nucleus (≈3α)

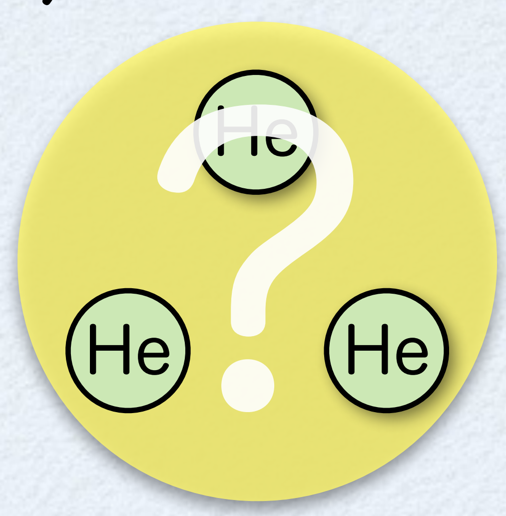
△ ^{4}He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted

1 was observed (1994)



E_b = 125.8 mK

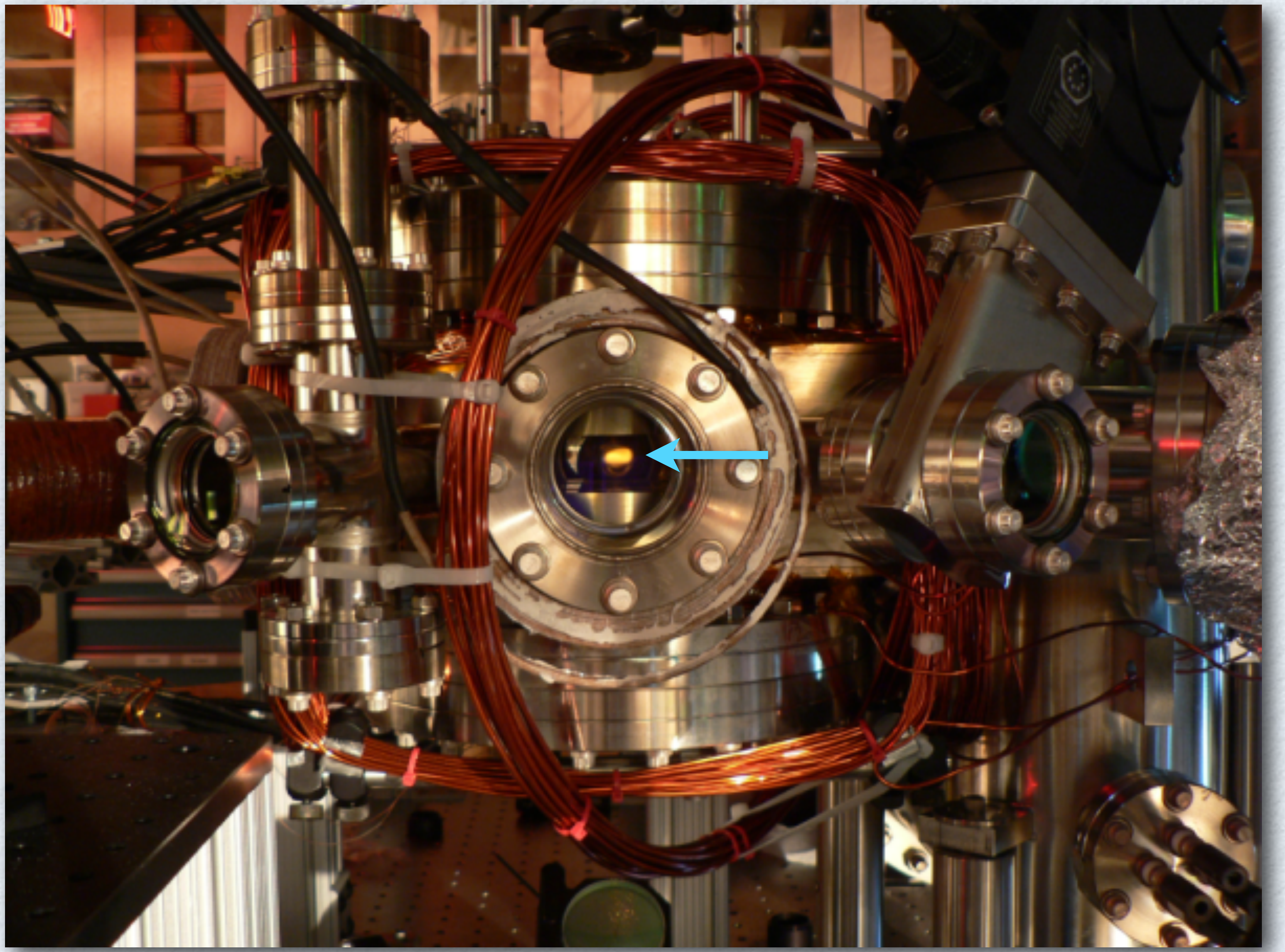


(E_b = 2.28 mK)

→ Ultracold atoms !

Ultracold atom experiments

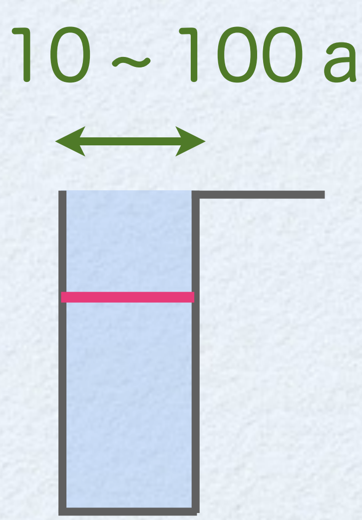
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

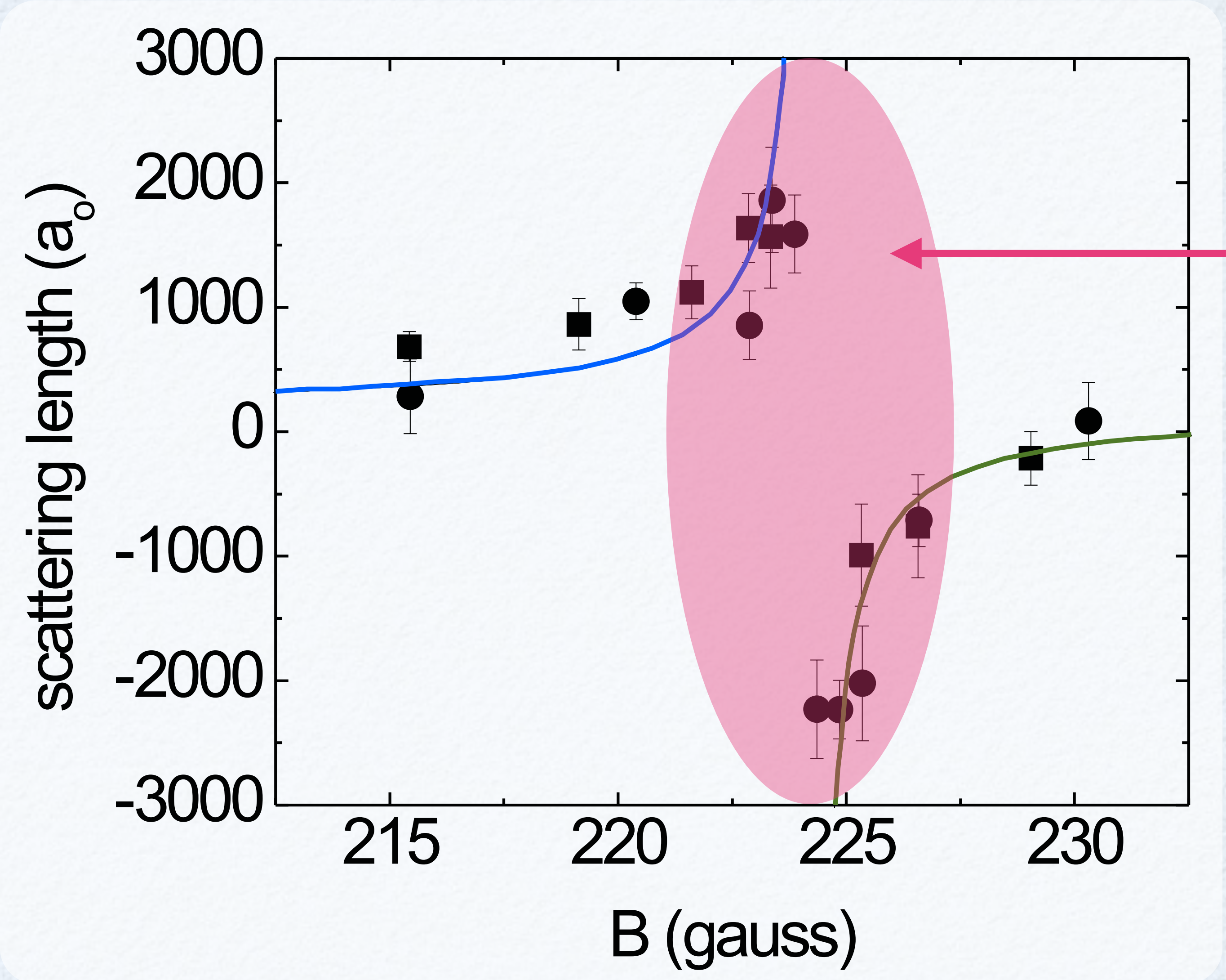


Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances

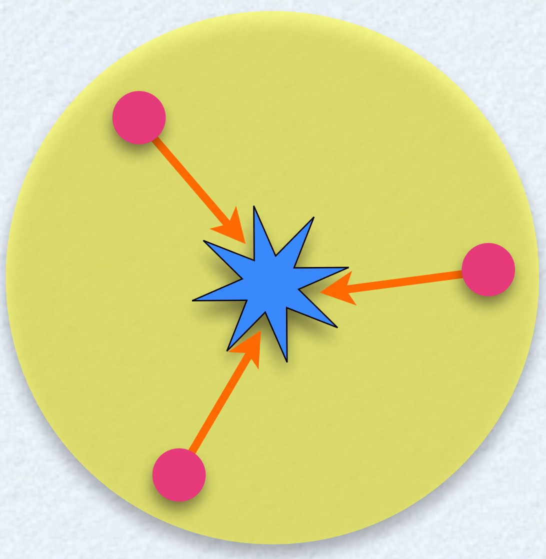




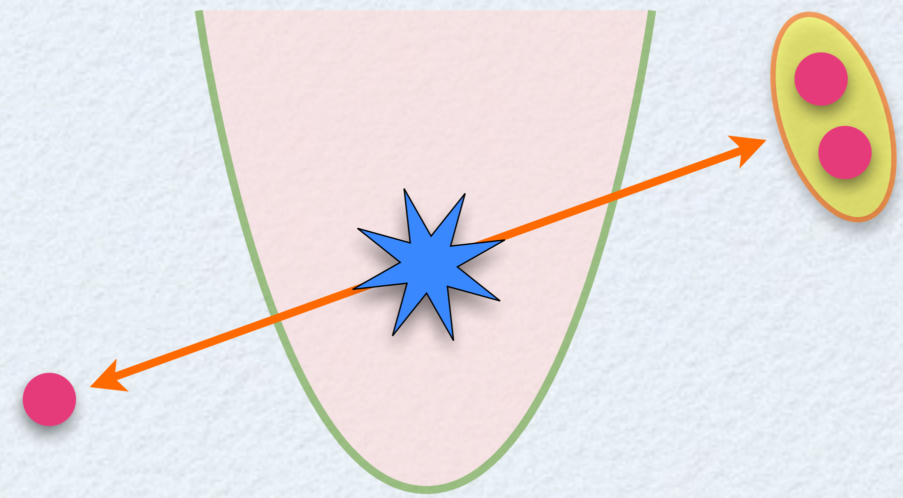
C.A. Regal & D.S. Jin
PRL90 (2003)

Ultracold atom experiments

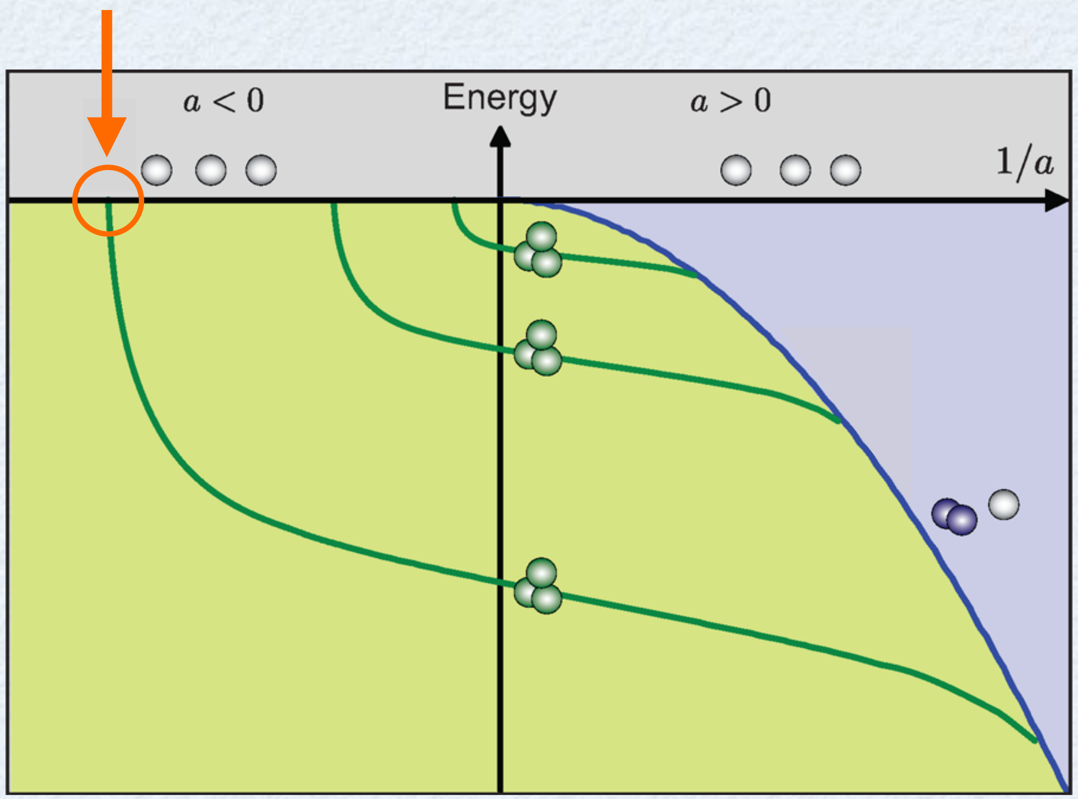
First experiment by Innsbruck group for ^{133}Cs (2006)

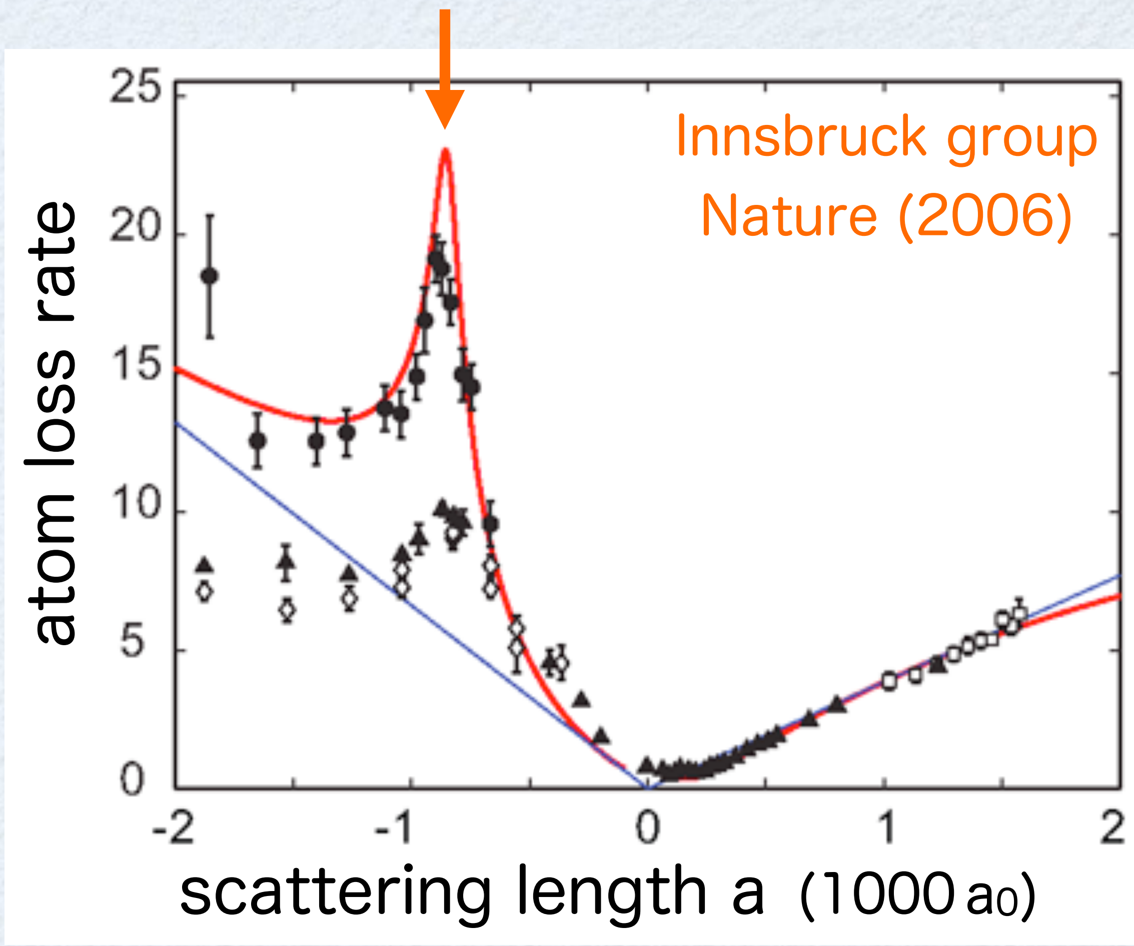


Trimer is unstable → atom loss

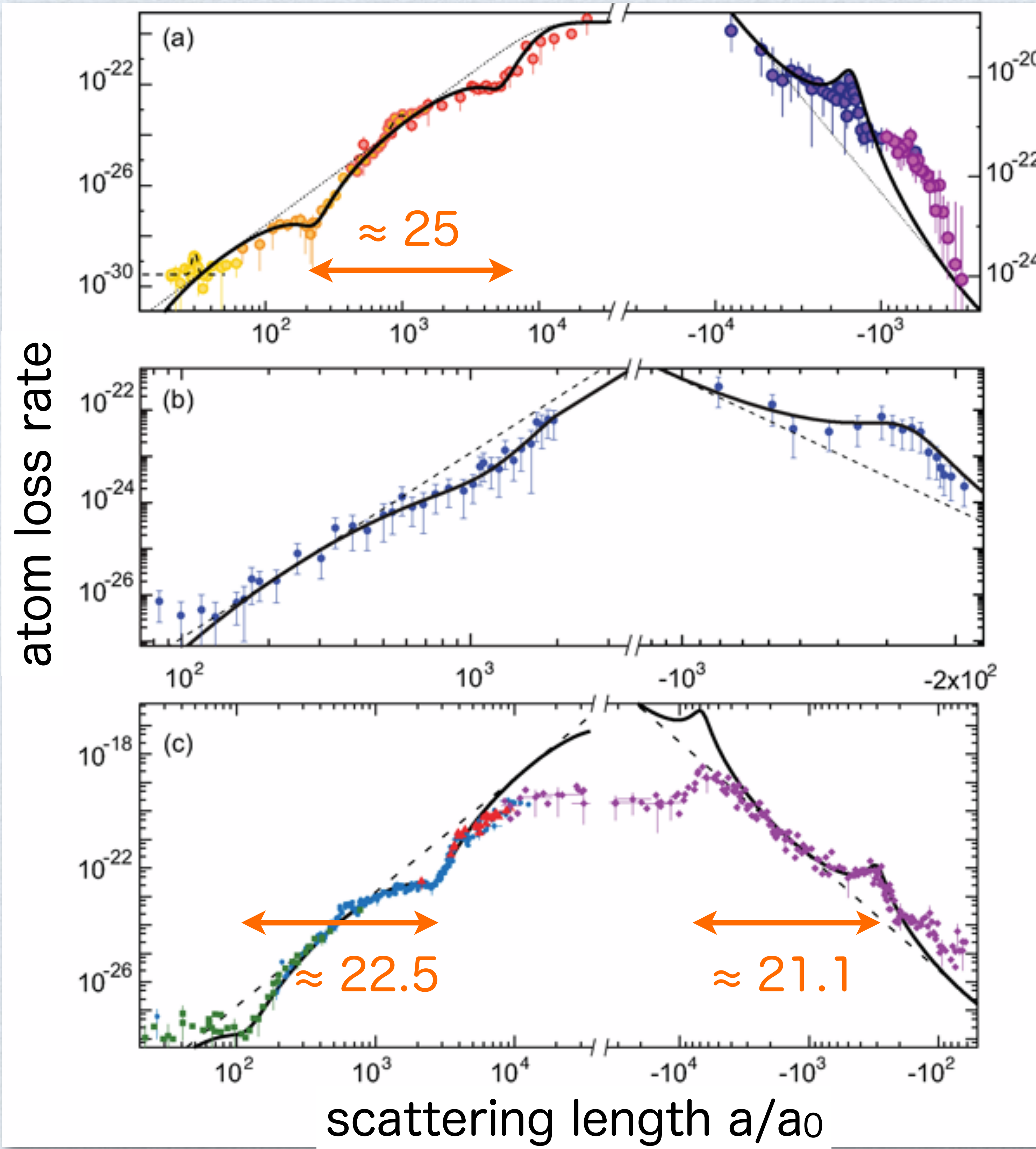


signature of trimer formation





Ultracold atom experiments



Florence group for ^{39}K (2009)

Bar-Ilan University for ^{7}Li (2009)

Rice University for ^{7}Li (2009)

→ Discrete scaling & Universality !

Efimov effect is "universal" ?

- Efimov effect is "universal"

  = appears regardless of microscopic details

  (physics technical term)

- Efimov effect is not "universal"

  universal = present or occurring everywhere

  (Merriam-Webster Online)

→ Can we find the Efimov effect in other physical systems ?

Efimov effect in quantum magnets

1. Universality in physics
2. What is the Efimov effect?
3. **Efimov effect in quantum magnets**
4. New progress: Super Efimov effect

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\right]$$

exchange anisotropy   single-ion anisotropy

Spin-boson correspondence

fully polarized state (B→∞)   No boson = vacuum

N spin-flips   N bosons = magnons

Quantum magnet

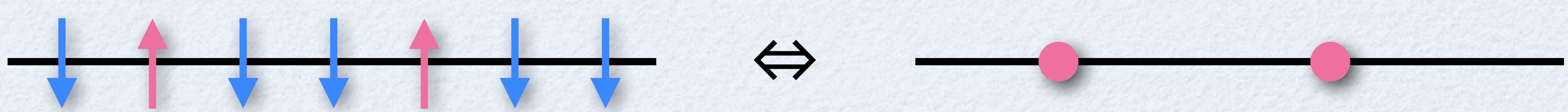
Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}\big(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z\big) + D(S_r^z)^2 - B S_r^z\Big]$$

xy-exchange coupling
⇔ **hopping**

single-ion anisotropy
⇔ **on-site attraction**

z-exchange coupling
⇔ **neighbor attraction**



N spin-flips ⇔ **N bosons = magnons**

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(JS_r^+S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - BS_r^z\Big]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction

Tune these couplings to induce
scattering resonance between two magnons
⇒ Three magnons show the Efimov effect

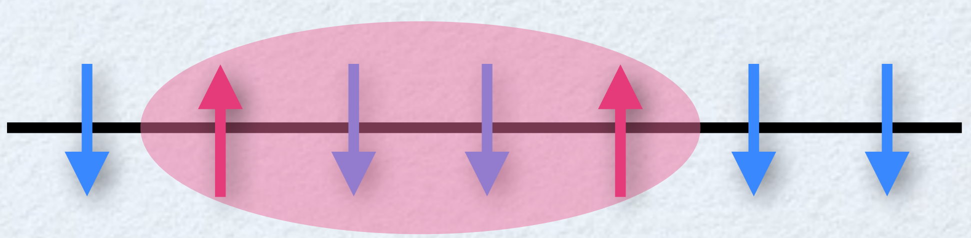
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

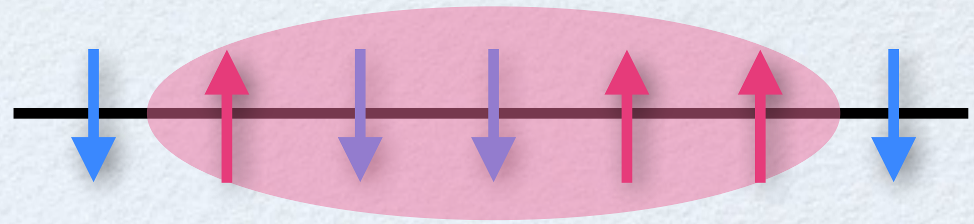
Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- Spin-1/2

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- Spin-1, D=0

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- Spin-1, $J_z=J>0$

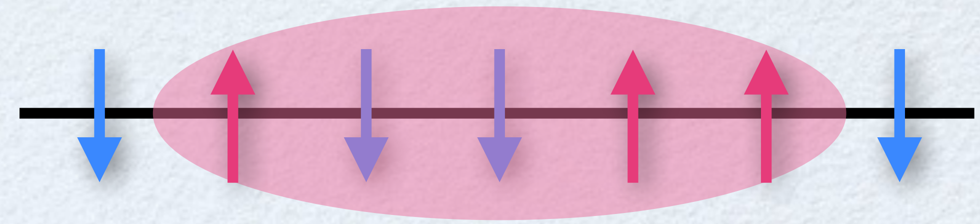
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- Spin-1, $J_z=J<0$

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

Universal scaling law by ~ 22.7 confirms they are Efimov states !

New progress

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in quantum magnets
4. **New progress: Super Efimov effect**

Super Efimov effect

PRL 110, 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

Yusuke Nishida,[1] Sergej Moroz,[2] and Dam Thanh Son[3]

[1]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[3]*Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA*

(Received 18 January 2013; published 4 June 2013)

We study a system of spinless fermions in two dimensions with a short-range interaction fine-tuned to a p-wave resonance. We show that three such fermions form an infinite tower of bound states of orbital angular momentum $\ell = \pm 1$ and their binding energies obey a universal doubly exponential scaling $E_3^{(n)} \propto \exp(-2e^{3\pi n/4+\theta})$ at large n. This "super Efimov effect" is found by a renormalization group analysis and confirmed by solving the bound state problem. We also provide an indication that there are $\ell = \pm 2$ four-body resonances associated with every three-body bound state at $E_4^{(n)} \propto \exp(-2e^{3\pi n/4+\theta-0.188})$. These universal few-body states may be observed in ultracold atom experiments and should be taken into account in future many-body studies of the system.

DOI: 10.1103/PhysRevLett.110.235301 PACS numbers: 67.85.Lm, 03.65.Ge, 05.30.Fk, 11.10.Hi

Introduction.—Recently topological superconductors have attracted great interest across many subfields in physics [1,2]. This is partially because vortices in topological superconductors bind zero-energy Majorana fermions and obey non-Abelian statistics, which can be of potential use for fault-tolerance topological quantum computation [3,4]. A canonical example of such topological superconductors is a p-wave paired state of spinless fermions in two dimensions [5], which is believed to be realized in Sr_2RuO_4 [6]. Previous mean-field studies revealed that a topological quantum phase transition takes place across a

of resonantly interacting fermions in two dimensions should be taken into account in future many-body studies beyond the mean-field approximation.

Renormalization group analysis.—The above predictions can be derived most conveniently by a renormalization group (RG) analysis. The most general Lagrangian density that includes up to marginal couplings consistent with rotation and parity symmetries is

$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a$$

Efimov vs. Super Efimov

Efimov effect

(Efimov in 1970)

- 3 bosons
- 3 dimensions
- s-wave resonance
- exponential scaling

$$\lambda_n \sim e^{\pi n}$$

Super Efimov effect

(Y.N, S.M, D.T.S in 2013)

- 3 fermions
- 2 dimensions
- p-wave resonance
- "doubly" exponential

$$\lambda_n \sim e^{e^{3\pi n/4}}$$

Are there other phenomena with doubly-exponential scaling ?

Hyper-inflation (economics)

東工大・高安研究室HP

2. ２重指数関数成長

２重指数関数的な変動は、ハイパーインフレーションと呼ばれる現象において一般的に観測される事が示されています。 これは、上述の指数関数成長時のXが一定値ではなく、時間の指数関数に従っている事を示しており、指数関数の肩に指数関数が乗っている事から、２重指数関数と呼ばれます。

Wikipedia

ハンガリー

ハンガリーでは第二次世界大戦後に激しいハイパーインフレが発生した。このときのインフレーションでは16年間で貨幣価値が1垓3000京分の1になったが、20桁以上のインフレーションは1946年前半の半年間に起きたものである。大戦後、1945年末まではインフレ率がほぼ一定であり、対ドルレートは指数関数的増大にとどまっていたが、1946年初頭からはインフレ率そのものが指数関数的に増大した。別の表現でいえば、物価が2倍になるのにかかる時間が、1か月、1週間、3日とだんだんと短くなっていったということである。当時を知るハンガリー人によると、一日で物価が2倍になる状況でも市場では紙幣が流通しており、現金を入手したものは皆、すぐに使ったという[26]。

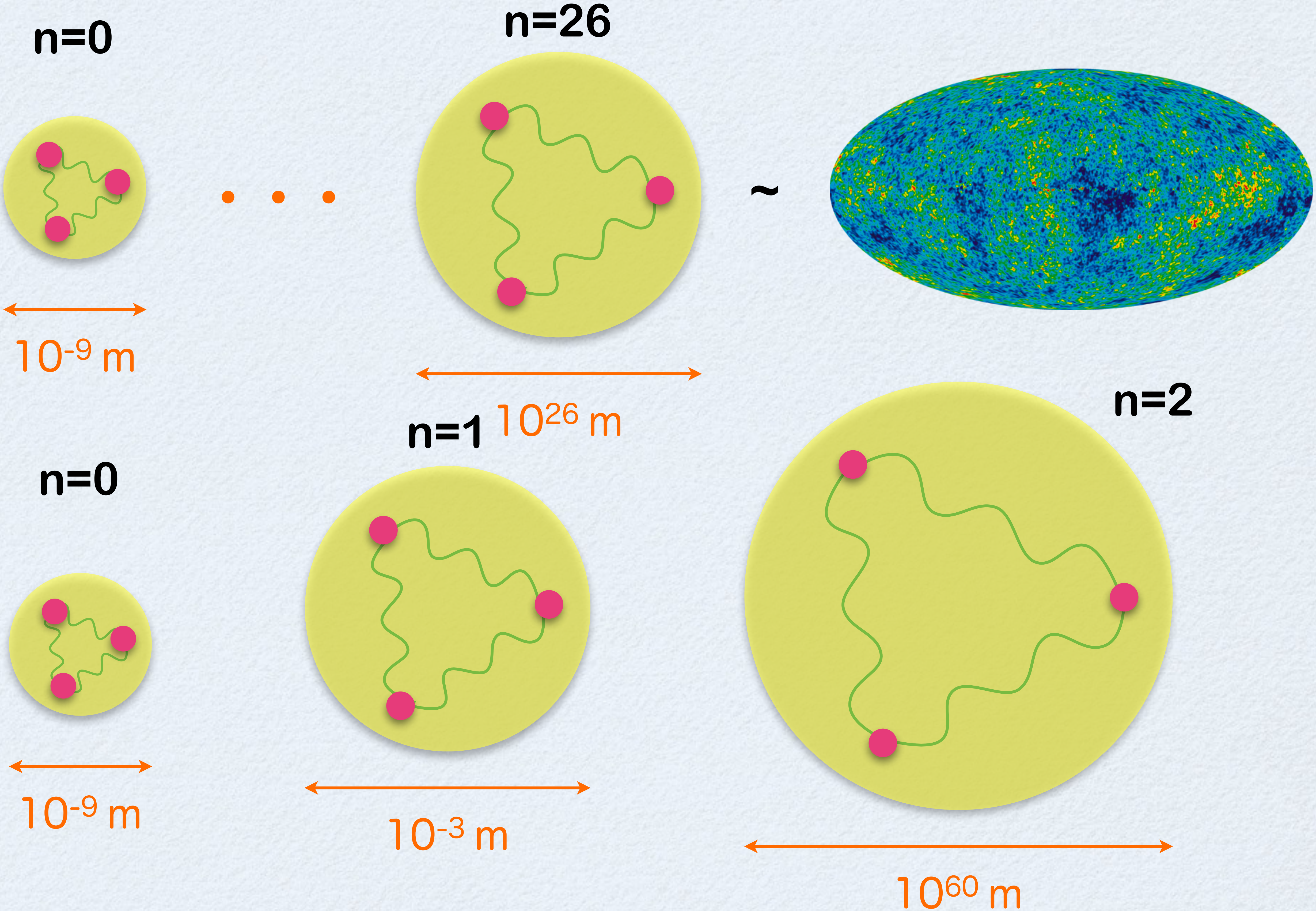
1946年に印刷された10垓ペンゲー紙幣（紙幣には10億兆と書かれている）が歴史上の最高額面紙幣であり（ただし、発行はされていない）、最悪のインフレーションとしてギネスブックに記録されている。

なお、実際に発行された最高額面紙幣は1垓ペンゲー紙幣（紙幣には1億兆と書かれている）である。

※1京は1兆の1万倍（10の16乗）、1垓は1京の1万倍（10の20乗）。

Are there other “physics” phenomena with doubly-exponential scaling ?

Efimov vs. Super Efimov



Summary

Efimov effect: universality, discrete scale inv, quantum anomaly, RG limit cycle

atomic physics

nuclear physics

condensed matter

✓ **Efimov effect in quantum magnets**

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ **New progress: Super Efimov effect**

Y.N, S.M, D.T.S, Phys.Rev.Lett.110, 235301 (2013)